

# Input/Output Machines



## Quick Review

This is an **Input/Output machine**.  
It can be used to make a growing pattern.



Each input is multiplied by 9 to get the output.

If you input 1, the output is 9.

If you input 2, the output is 18.

Input	Output
1	9
2	18
3	27
4	36
5	45

The pattern rule for the output is:

Start at 9. Add 9 each time.

## Try These

1. Complete the table of values for each Input/Output machine.

a)



Input	Output
17	
16	
15	
14	
13	
12	
11	

b)



Input	Output
40	
36	
32	
28	
24	
20	
16	

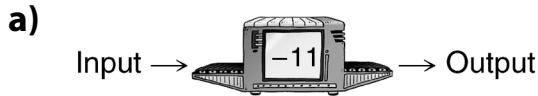
2. Look at the tables of values in question 1. Write the pattern rule for each group of terms.

a) the output numbers in part a) \_\_\_\_\_

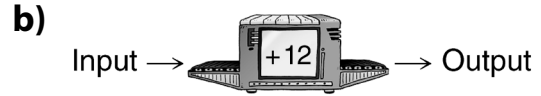
b) the input numbers in part b) \_\_\_\_\_

## Practice

1. Complete the table of values for each Input/Output machine.

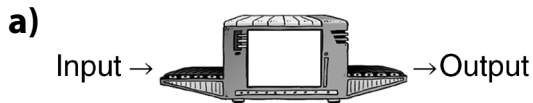


Input	Output
93	
90	
87	
84	
81	



Input	Output
305	
310	
315	
320	
325	

2. Look at the tables of values. Write the number and the operation in each machine.



Input	Output
840	42
800	40
760	38
720	36
680	34



Input	Output
11	143
20	260
29	377
38	494
47	611

## Stretch Your Thinking

The table of values shows the Input/Output from a machine.

- Write the number and operation for the machine. \_\_\_\_\_
- Write the pattern rule for the input numbers.  
\_\_\_\_\_
- Write the pattern rule for the output numbers. \_\_\_\_\_

Input	Output
3456	1152
3531	1177
3606	1202
3681	1227
3756	1252

# Patterns from Tables



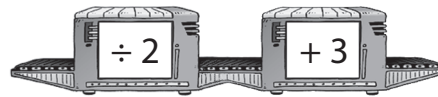
## Quick Review

This Input/Output machine divides each input by 2, then adds 3.

The pattern rule that relates the input to the output is: Divide the input by 2. Then add 3.

We can use this rule to predict the output for any input.

For an input of 70, the output is:  
 $70 \div 2 + 3 = 38$



Input	Output
20	13
30	18
40	23
50	28
60	33

## Try These

- Each table of values shows the input and output from a machine with 1 operation. Write the number and the operation in each machine.

a)



Input	Output
2	4
4	8
6	12
8	16
10	20

b)



Input	Output
24	6
20	5
16	4
12	3
8	2

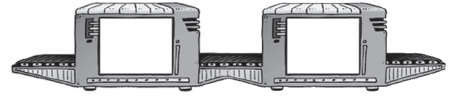
- Write the pattern rule that relates the input to the output for each table of values in question 1.

a) \_\_\_\_\_

b) \_\_\_\_\_

## Practice

1. Each table shows the input and output from a machine with 2 operations.



For each table, write the numbers and the operations in the machine.

a)

Input	Output
4	25
5	32
6	39
7	46

b)

Input	Output
50	20
55	22
60	24
65	26

c)

Input	Output
7	26
8	28
9	30
10	32

2. Write the pattern rule that relates the input to the output for each table in question 1.

- a) \_\_\_\_\_  
 b) \_\_\_\_\_  
 c) \_\_\_\_\_

3. This table shows the input and output from a machine with 2 operations.

- a) Write the numbers and the operations in the machine.

\_\_\_\_\_

- b) Write the next 3 input and output numbers.

- c) Predict the output when the input is 100.

\_\_\_\_\_

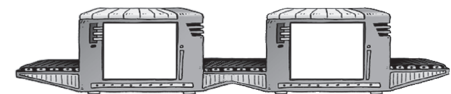
Input	Output
25	15
30	18
35	21

## Stretch Your Thinking

The first 5 input numbers for the machine are:  
2527, 2577, 2627, 2677, and 2727.

The first 5 output numbers for the machine are:  
5061, 5161, 5261, 5361, and 5461.

Write the numbers and the operations in the machine.



# Using Variables to Describe Patterns



## Quick Review

The pattern rule for the output is:

Start at 5. Add 2 each time.

This suggests the input numbers are multiplied by 2.

Multiply input 3 by 2:  $3 \times 2 = 6$

To get output 9, add 3.

The pattern rule that relates the input to the output is: Multiply by 2. Then add 3.

We can use a variable in an expression to represent this rule.

Let the letter  $n$  represent any input number.

Then, the expression  $2n + 3$  relates the input to the output.

Input	Output
1	5
2	7
3	9
4	11
5	13

Input	Output
1	$2 \times 1 + 3 = 5$
2	$2 \times 2 + 3 = 7$
3	$2 \times 3 + 3 = 9$
4	$2 \times 4 + 3 = 11$
5	$2 \times 5 + 3 = 13$
⋮	⋮
$n$	$2 \times n + 3$

## Try These

- Complete each table of values, then write an expression that relates the input to the output.

a)

Input	Output
1	3
2	8
3	13
4	18
5	23
6	
7	
8	
9	

b)

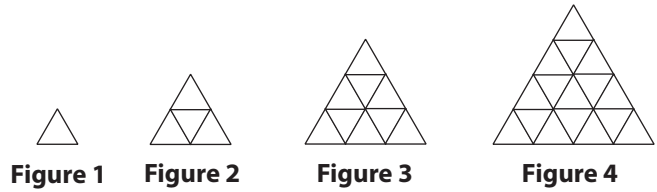
Input	Output
1	9
2	14
3	19
4	24
5	29
6	
7	

c)

Input	Output
0	4
1	10
2	16
3	22
4	28
5	

## Practice

1. Here is a pattern of triangles.



- a) Complete the table.
- b) Write the pattern rule.
- \_\_\_\_\_
- c) Write an expression for the pattern.
- \_\_\_\_\_
- d) Find the number of triangles in the 8th figure.
- \_\_\_\_\_

Figure	Number of Triangles
1	
2	
3	
4	

2. For each table of values, write an expression to represent the pattern.

a)

Input	Output
1	1
2	5
3	9
4	13

\_\_\_\_\_

b)

Input	Output
2	4
3	9
4	14
5	19

\_\_\_\_\_

## Stretch Your Thinking

- a) Use the expression  $7n + 10$  to complete the table.
- b) Write and solve a story problem that matches the pattern.
- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_

Number	Amount (\$)
0	
1	
2	
3	
4	

# Plotting Points on a Coordinate Grid



## Quick Review

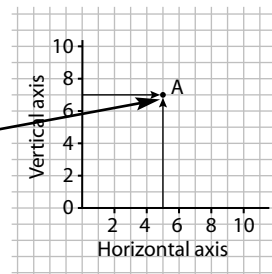
- We use an **ordered pair** to describe the **coordinates** of a point on a grid.

The coordinates of point A are (5, 7).

The **origin** is the point where the horizontal and vertical axes meet.

In an ordered pair:

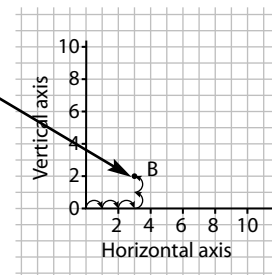
- The first number tells the horizontal distance from the origin.
- The second number tells the vertical distance from the origin.



- The coordinates of point B are (3, 2).

To **plot** point B:

Start at 0, move 3 squares right, then move 2 squares up.



## Try These

1. a) Name the letter on the grid represented by each ordered pair.

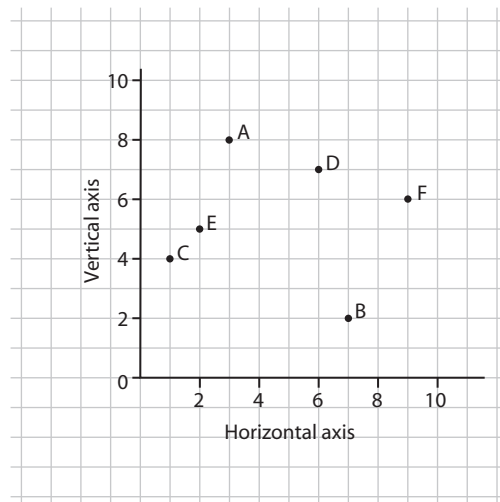
(2, 5) \_\_\_\_\_ (6, 7) \_\_\_\_\_ (1, 4) \_\_\_\_\_

(9, 6) \_\_\_\_\_ (7, 2) \_\_\_\_\_ (3, 8) \_\_\_\_\_

- b) Plot each point on the grid.

G(5, 4), H(10, 10), I(0, 9),

J(0, 2), K(8, 1), L(10, 4)



## Practice

1. Plot each set of ordered pairs on the coordinate grid.  
Join the points in order.  
Join the last point to the first point.  
Name each polygon you have drawn.

A:  $(8, 6), (6, 6), (6, 8), (8, 8)$

\_\_\_\_\_

B:  $(0, 3), (4, 0), (6, 0), (2, 3)$

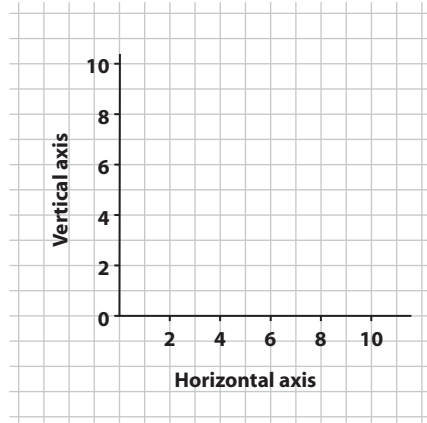
\_\_\_\_\_

C:  $(1, 6), (1, 10), (4, 10), (4, 6)$

\_\_\_\_\_

D:  $(7, 1), (6, 3), (8, 5), (10, 3), (9, 1)$

\_\_\_\_\_

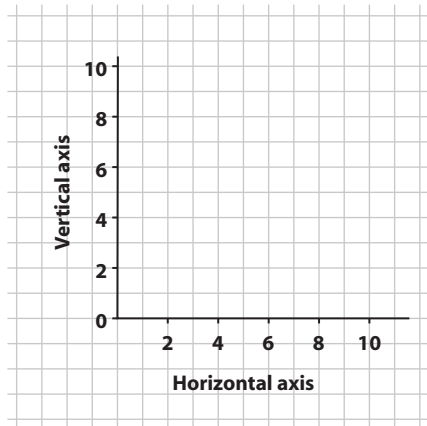


2. Plot 6 points on the grid.  
Label the points A to F.  
Record the ordered pairs.

A: \_\_\_\_\_ B: \_\_\_\_\_

C: \_\_\_\_\_ D: \_\_\_\_\_

E: \_\_\_\_\_ F: \_\_\_\_\_



## Stretch Your Thinking

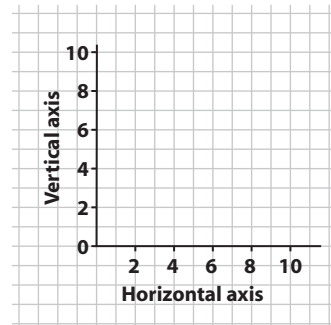
- $(2, 5)$  and  $(7, 5)$  are 2 vertices of a parallelogram with area 10 square units.  
Plot the points for the 2 given vertices.  
What are the coordinates of the other vertices?  
Give as many answers as you can.

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_





# Drawing the Graph of a Pattern



## Quick Review

Here are some ways to represent a pattern.

- Model the pattern on grid paper.

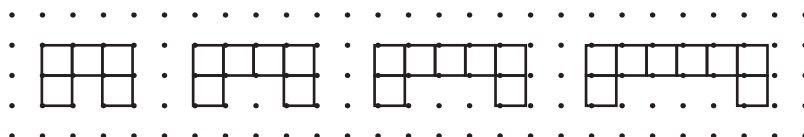


Figure 1

Figure 2

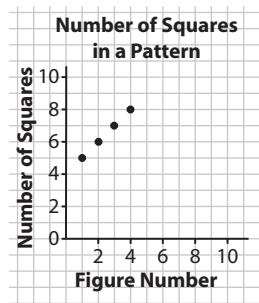
Figure 3

Figure 4

- Make a table.

Figure Number	Number of Squares	Ordered Pair
1	5	(1, 5)
2	6	(2, 6)
3	7	(3, 7)
4	8	(4, 8)

- Draw a graph.



## Try These

- Henry made this pattern.

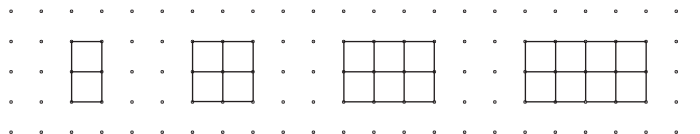


Figure 1

Figure 2

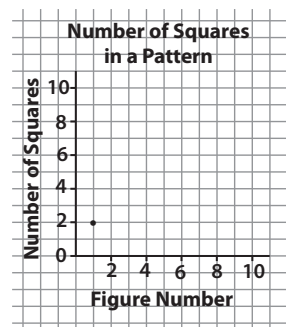
Figure 3

Figure 4

- Complete the table.

Figure Number	Number of Squares	Ordered Pair
1	2	(1, 2)

- Graph the pattern



## Practice

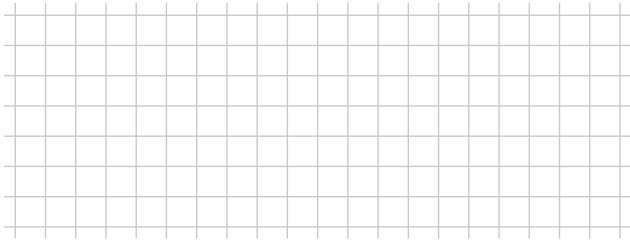
1. a) Describe the relationship shown in the table.

<b>Figure Number</b>	1	2	3	4	5
<b>Number of Squares</b>	1	3	5	7	9

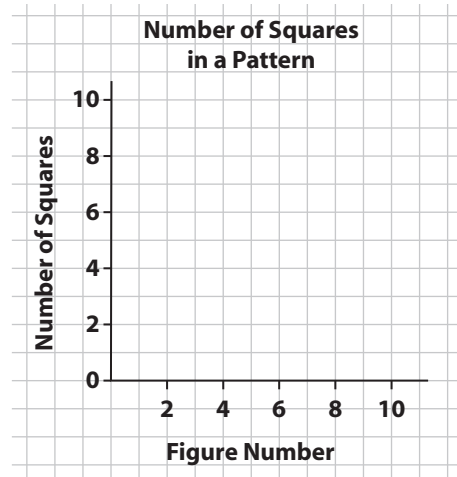
\_\_\_\_\_

\_\_\_\_\_

- b) Draw squares on the grid to model the pattern.



- c) Graph the pattern.
- d) How many squares are needed for Figure 10?



- e) Which figure has 29 squares?
- f) Which figure has 51 squares?

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

2. Draw a pattern to model the data in the table.

<b>Figure Number</b>	1	2	3	4
<b>Number of Triangles</b>	1	2	4	8



## Stretch Your Thinking

Use the table in question 2.

How many triangles are in Figure 10? \_\_\_\_\_

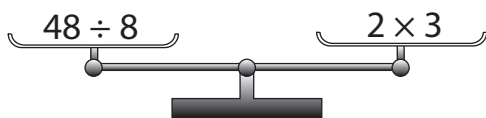
Which figure has 8192 triangles? \_\_\_\_\_

# Understanding Equality

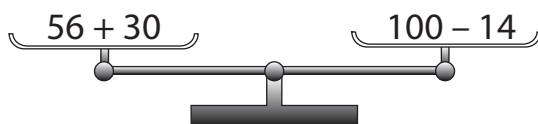


## Quick Review

- ▶ Each of these scales is balanced.  
The expression in one pan is equal to the expression in the other pan.



$48 \div 8 = 6$  and  
 $2 \times 3 = 6$   
 So,  $48 \div 8 = 2 \times 3$



$56 + 30 = 86$  and  
 $100 - 14 = 86$   
 So,  $56 + 30 = 100 - 14$

- ▶ When we add 2 numbers, their order does not affect the sum.  
This is called the **commutative property of addition**.  
 $7 + 5 = 5 + 7$   
 $a + b = b + a$
- ▶ When we multiply 2 numbers, their order does not affect the product.  
This is called the **commutative property of multiplication**.  
 $6 \times 3 = 3 \times 6$   
 $a \times b = b \times a$

## Try These

1. Rewrite each expression using a commutative property.

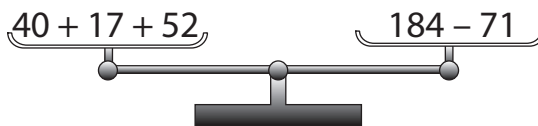
a)  $9 + 6$  \_\_\_\_\_

b)  $7 \times 4$  \_\_\_\_\_

c)  $751 + 242$  \_\_\_\_\_

d)  $27 \times 8$  \_\_\_\_\_

2. Are these scales balanced?  
How do you know?



\_\_\_\_\_

\_\_\_\_\_

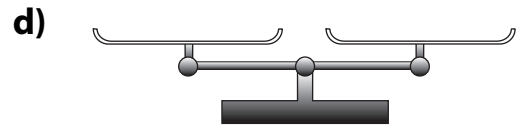
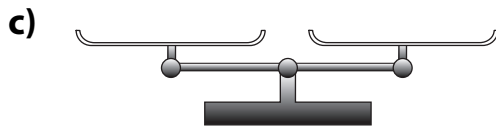
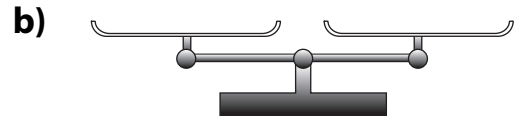
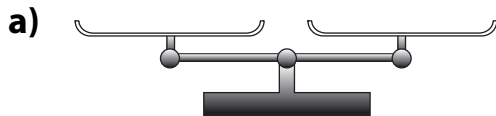
## Practice

1. Work with a partner.

Write an expression in one pan of a balance scale.

Your partner writes a different expression to balance the scale.

Continue with each balance scale. Switch roles at each turn.



2. Draw a line to join pairs of expressions that balance.

a)

Expressions	
$8 \times 9$	$2 \times 53$
$522 \div 9$	$24 + 76$
$75 + 31$	$314 - 242$
$10 \times 10$	$29 \times 2$

b)

Expressions	
$764 - 320$	$4000 - 48$
$76 \times 52$	$18 \div 3$
$36 \div 6$	$5 \times 25$
$52 + 73$	$4 \times 111$

## Stretch Your Thinking

Write 3 equal expressions for each expression below.

a)  $57 + 46 - 31$

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

b)  $45 \times 2 + 17$

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

c)  $425 \div 5 + 36$

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

# Keeping Equations Balanced



## Quick Review

- We can model this equation with counters:  $3 + 3 = 4 + 2$

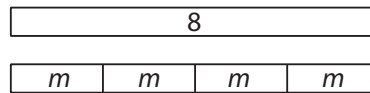


Multiply each side by 2.  
 $6 \times 2 = 6 \times 2$



When each side of an equation is changed in the same way, the values remain equal. This is called the **preservation of equality**.

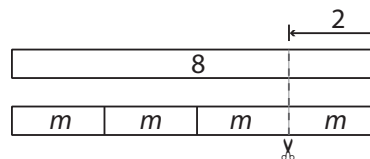
- Suppose we know  $8 = 4m$ . We can model this equation with paper strips.



To preserve the equality, we can subtract the same number from each side.

$$8 - 2 = 4m - 2$$

So,  $8 - 2 = 4m - 2$  is an **equivalent form** of  $8 = 4m$ .

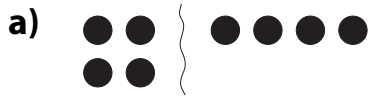


## Try These

1. Model each equation with counters.  
 Use counters to model the preservation of equality. Record your work.
  - a)  $3 + 2 = 1 + 4$
  - b)  $18 \div 3 = 3 \times 2$

## Practice

1. Use addition to preserve the equality of each equation.



\_\_\_\_\_



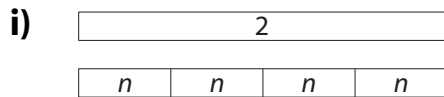
\_\_\_\_\_

2. Use subtraction to preserve the equality of each equation in question 1.

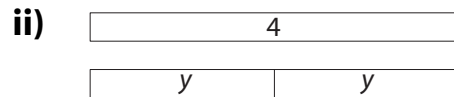
a) \_\_\_\_\_

b) \_\_\_\_\_

3. a) Write an equation for each diagram.



\_\_\_\_\_



\_\_\_\_\_

b) Use multiplication to preserve the equality of each equation.  
Record your work.

i) \_\_\_\_\_

ii) \_\_\_\_\_

## Stretch Your Thinking

Apply the preservation of equality. Write an equivalent form of the equation.  
Use a different operation for each part.

a)  $5y = 20$

\_\_\_\_\_

b)  $20 \div 5 = 8 - 4$

\_\_\_\_\_

c)  $8 \times 6 = 12 \times 4$

\_\_\_\_\_

d)  $5 + 19 = 6s$

\_\_\_\_\_