UNIT 4

Circles and Area

Look at these pictures.
What shapes do you see?

- What is a circle?
- Where do you see circles?
- What do you know about a circle?
- What might be useful to know about a circle?
  A parallelogram?
  A triangle?

What You'll Learn

- Investigate and explain the relationships among the radius, diameter, and circumference of a circle.
- Determine the sum of the central angles of a circle.
- Construct circles and solve problems involving circles.
- Develop formulas to find the areas of a parallelogram, a triangle, and a circle.
- Draw, label, and interpret circle graphs.

Why It's Important

- The ability to measure circles, triangles, and parallelograms is an important skill.
  These shapes are used in design, architecture, and construction.

Key Words

- radius, radii
- diameter
- circumference
- π
- irrational number
- base
- height
- circle graph
- sector
- legend
- percent circle
- central angle
- sector angle
- pie chart

4.1 Investigating Circles

Focus Measure radius and diameter and discover their relationship.

Explore

You will need circular objects, a compass, and a ruler.

➤ Use a compass. Draw a large circle. Use a ruler.
Draw a line segment that joins two points on the circle. Measure the line segment.
Label the line segment with its length.
Draw and measure other segments that join two points on the circle.
Find the longest segment in the circle.
How many other segments can you draw with this length? Repeat the activity for other circles.

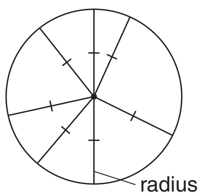
➤ Trace a circular object. Cut out the circle.
How many ways can you find the centre of the circle?
Measure the distance from the centre to the circle.
Measure the distance across the circle, through its centre.
Record the measurements in a table.
Repeat the activity with other circular objects.
What pattern do you see in your results?

Reflect & Share

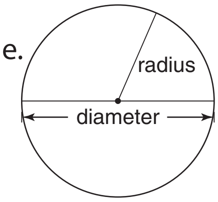
Compare your results with those of another pair of classmates.
Where is the longest segment in any circle?
What relationship did you find between the distance across a circle through its centre, and the distance from the centre to the circle?

Connect

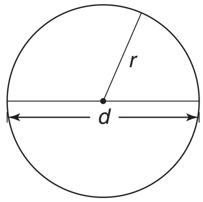
All points on a circle are the same distance from the centre of the circle.
This distance is the **radius** of the circle.



The longest line segment in any circle is the **diameter** of the circle.
The diameter passes through the centre of the circle.
The radius is one-half the length of the diameter.
The diameter is two times the length of the radius.



Let r represent the radius, and d the diameter.
Then the relationship between the radius and diameter of a circle is:
$r = d \div 2$, which can be written as $r = \frac{d}{2}$
And, $d = 2r$



The plural of *radius* is *radii*; that is, one radius, two or more radii.

Example

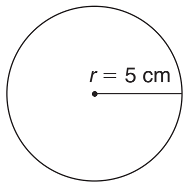
Use a compass. Construct a circle with:

a) radius 5 cm **b)** diameter 10 cm

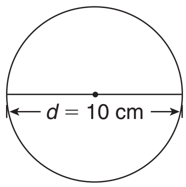
What do you notice about the circles you constructed?

A Solution

a) Draw a line segment with length 5 cm.
Place the compass point at one end.
Place the pencil point at the other end.
Draw a circle.



b) Draw a line segment with length 10 cm.
Use a ruler to find its midpoint.
Place the compass point at the midpoint.
Place the pencil point at one end of the segment.
Draw a circle.



The two circles are congruent.
A circle with radius 5 cm has diameter 10 cm.

Recall that congruent shapes are identical.

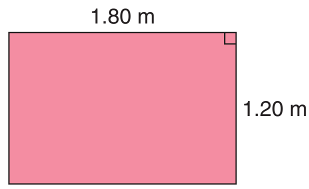
Practice

1. Use a compass.
Draw a circle with each radius.
a) 6 cm **b)** 8 cm
Label the radius, then find the diameter.

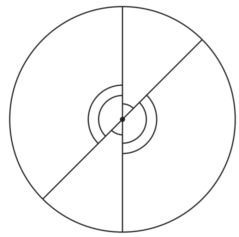
2. Draw a circle with each radius without using a compass.
   a) 7 cm b) 4 cm
   Label the radius, then find the diameter.
   Explain the method you used to draw the circles.
   What are the disadvantages of not using a compass?

3. a) A circle has diameter 3.8 cm. What is the radius?
   b) A circle has radius 7.5 cm. What is the diameter?

4. A circular tabletop is to be cut from a rectangular piece of wood that measures 1.20 m by 1.80 m.
   What is the radius of the largest tabletop that could be cut?
   Justify your answer. Include a sketch.



5. a) Use a compass. Draw a circle. Draw 2 different diameters.
   b) Use a protractor. Measure the angles at the centre of the circle.
   c) Find the sum of the angles.
   d) Repeat parts a to c for 3 different circles.
   What do you notice about the sum of the angles in each circle?



6. A glass has a circular base with radius 3.5 cm.
   A rectangular tray has dimensions 40 cm by 25 cm.
   How many glasses will fit on the tray?
   What assumptions did you make?

7. **Assessment Focus** Your teacher will give you a large copy of this logo.
   Find the radius and diameter of each circle in this logo. Show your work.

This is the logo for the Aboriginal Health Department of the Vancouver Island Health Authority.

8. **Take It Further** A circular area of grass needs watering.
   A rotating sprinkler is to be placed at the centre of the circle.
   Explain how you would locate the centre of the circle.
   Include a diagram in your explanation.

Reflect

How are the diameter and radius of a circle related?
Include examples in your explanation.

4.2 Circumference of a Circle

Focus Investigate the relationship between the circumference and diameter of a circle.

Explore

You will need 3 circular objects of different sizes, string, and a ruler.

➤ Each of you chooses one of the objects.
Use string to measure the distance around it.
Measure the radius and diameter of the object.
Record these measures.

➤ Repeat the activity until each of you has measured all 3 objects.
Compare your results.
If your measures are the same, record them in a table.
If your measures for any object are different, measure again to check.
When you agree upon the measures, record them in the table.

Object	Distance Around (cm)	Radius (cm)	Diameter (cm)
Can			

➤ What patterns do you see in the table?
How is the diameter related to the distance around?
How is the radius related to the distance around?

➤ For each object, calculate:
- distance around ÷ diameter
- distance around ÷ radius

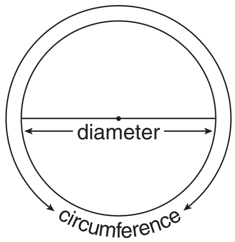
What do you notice?
Does the size of the circle affect your answers? Explain.

Reflect & Share

Compare your results with those of another group.
Suppose you know the distance around a circle.
How can you find its diameter?

Connect

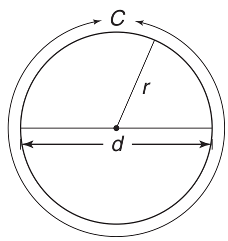
The distance around a circle is its **circumference**.
For any circle, the circumference, C, divided by the diameter, d, is approximately 3.
Circumference ÷ diameter ≐ 3, or $\frac{C}{d} \doteq 3$



The circumference of a circle is also the perimeter of the circle.

For any circle, the ratio $\frac{C}{d} = \pi$
The symbol **π** is a Greek letter that we read as "pi."
$\pi = 3.141\,592\,653\,589\ldots$, or $\pi \doteq 3.14$

π is a decimal that never repeats and never terminates.
π cannot be written as a fraction.
For this reason, we call π an **irrational number**.



So, the circumference is π multiplied by d.
We write: $C = \pi d$

Since the diameter is twice the radius,
the circumference is also π multiplied by $2r$.
We write: $C = \pi \times 2r$, or $C = 2\pi r$

When we know the radius or diameter of a circle, we can use one of the formulas above to find the circumference of the circle.

The face of a toonie has radius 1.4 cm.
- To find the diameter of the face:
  The diameter $d = 2r$, where r is the radius
  Substitute: $r = 1.4$
  $d = 2 \times 1.4$
  $= 2.8$
  The diameter is 2.8 cm.

The circumference is a length, so its units are units of length such as centimetres, metres, or millimetres.

- To find the circumference of the face:

$C = \pi d$	OR $C = 2\pi r$
Substitute: $d = 2.8$	Substitute: $r = 1.4$
$C = \pi \times 2.8$	$C = 2 \times \pi \times 1.4$
$\doteq 8.796$	$\doteq 8.796$
$\doteq 8.8$	$\doteq 8.8$

  The circumference is 8.8 cm, to one decimal place.

Use the π key on your calculator. If the calculator does not have a π key, use 3.14 instead.

- We can estimate to check if the answer is reasonable.
  The circumference is approximately 3 times the diameter:
  $3 \times 2.8\text{ cm} \doteq 3 \times 3\text{ cm}$
  $= 9\text{ cm}$
  The circumference is approximately 9 cm.
  The calculated answer is 8.8 cm, so this answer is reasonable.

When we know the circumference, we can use a formula to find the diameter.
Use the formula $C = \pi d$.
To isolate d, divide each side by π.

$$\frac{C}{\pi} = \frac{\pi d}{\pi}$$

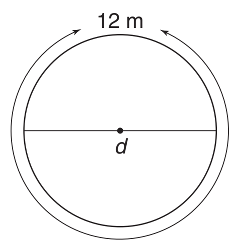
$$\frac{C}{\pi} = d$$

So, $d = \frac{C}{\pi}$

Example

An above-ground circular swimming pool has circumference 12 m.
Calculate the diameter and radius of the pool.
Give the answers to two decimal places.
Estimate to check the answers are reasonable.

A Solution



The diameter is: $d = \frac{C}{\pi}$
Substitute: $C = 12$

$d = \frac{12}{\pi}$

$= 3.8197\ldots$

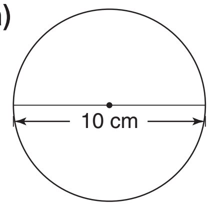
Use a calculator.
Do not clear your calculator.

The radius is $\frac{1}{2}$ the diameter, or $r = d \div 2$.
Divide the number in the calculator display by 2.
$r \doteq 1.9099$
The diameter is 3.82 m to two decimal places.
The radius is 1.91 m to two decimal places.

Since the circumference is approximately 3 times the diameter,
the diameter is about $\frac{1}{3}$ the circumference.
One-third of 12 m is 4 m. So, the diameter is about 4 m.
The radius is $\frac{1}{2}$ the diameter. One-half of 4 m is 2 m.
So, the radius of the pool is about 2 m.
Since the calculated answers are close to the estimates, the answers are reasonable.

Practice

1. Calculate the circumference of each circle.
Give the answers to two decimal places.
Estimate to check the answers are reasonable.

a)


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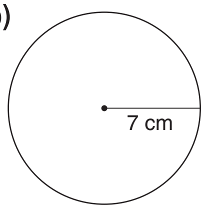

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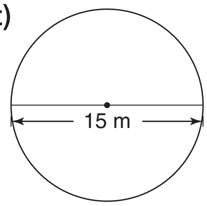
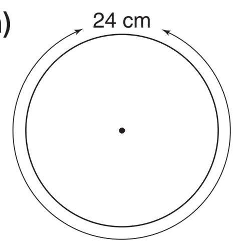


2. Calculate the diameter and radius of each circle.
Give the answers to two decimal places.
Estimate to check the answers are reasonable.

a)


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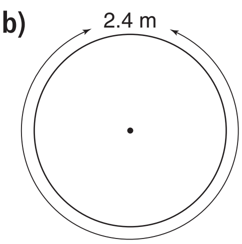

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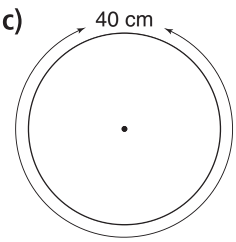


3. When you estimate to check the circumference, you use 3 instead of π.
Is the estimated circumference greater than or less than
the actual circumference?
Why do you think so?

4. A circular garden has diameter 2.4 m.
a) The garden is to be enclosed with plastic edging.
How much edging is needed?
b) The edging costs $4.53/m.
What is the cost to edge the garden?

5. a) Suppose you double the diameter of a circle.
   What happens to the circumference?
   b) Suppose you triple the diameter of a circle.
   What happens to the circumference?
   Show your work.

6. A carpenter is making a circular tabletop with circumference 4.5 m.
   What is the radius of the tabletop in centimetres?

Recall: 1 m = 100 cm

7. Can you draw a circle with circumference 33 cm?
   If you can, draw the circle and explain how you know its circumference is correct.
   If you cannot, explain why it is not possible.

8. **Assessment Focus** A bicycle tire has a spot of wet paint on it.
   The radius of the tire is 46 cm.
   Every time the wheel turns, the paint marks the ground.
   a) What pattern will the paint make on the ground as the bicycle moves?
   b) How far will the bicycle have travelled between two consecutive paint marks on the ground?
   c) Assume the paint continues to mark the ground.
   How many times will the paint mark the ground when the bicycle travels 1 km?
   Show your work.

9. **Take It Further** Suppose a metal ring could be placed around Earth at the equator.
   a) The radius of Earth is 6378.1 km. How long is the metal ring?
   b) Suppose the length of the metal ring is increased by 1 km.
   Would you be able to crawl under the ring, walk under the ring, or drive a school bus under the ring?
   Explain how you know.

Reflect

What is π?
How is it related to the circumference, diameter, and radius of a circle?

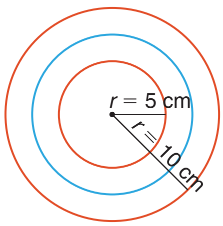
Mid-Unit Review

LESSON

4.1

1. **a)** Use a compass.
Draw a circle with radius 3 cm.
b) Do not use a compass.
Draw a circle with radius 7 cm.
The circle should have the same centre as the circle in part a.

2. Two circles have the same centre. Their radii are 5 cm and 10 cm. Another circle lies between these circles. Give two possible diameters for this circle.



3. Find the radius of a circle with each diameter.
a) 7.8 cm **b)** 8.2 cm
c) 10 cm **d)** 25 cm

4. Is it possible to draw two different circles with the same radius and diameter? Why or why not?

4.2

5. Calculate the circumference of each circle. Give the answers to two decimal places. Estimate to check your answers are reasonable.

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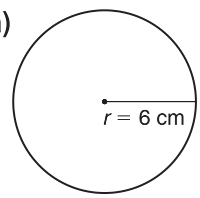

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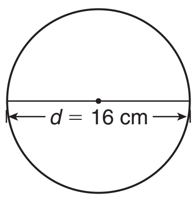


6. **a)** Calculate the circumference of each object.
i) A wheelchair wheel with diameter 66 cm
ii) A tire with radius 37 cm
iii) A hula-hoop with diameter 60 cm
b) Which object has the greatest circumference? How could you tell without calculating the circumference of each object?

7. Suppose the circumference of a circular pond is 76.6 m. What is its diameter?

8. Find the radius of a circle with each circumference. Give your answers to one decimal place.
a) 256 cm **b)** 113 cm **c)** 45 cm

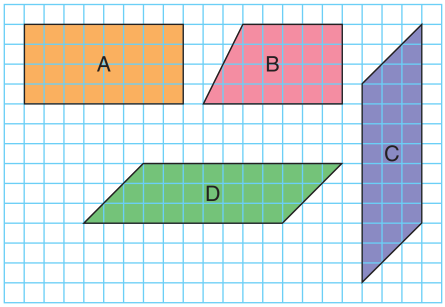
9. An auger is used to drill a hole in the ice, for ice fishing. The diameter of the hole is 25 cm. What is the circumference of the hole?

4.3 Area of a Parallelogram

Focus Develop a formula to find the area of a parallelogram.

Which of these shapes are parallelograms?
How do you know?

How are Shapes C and D alike?
How are they different?

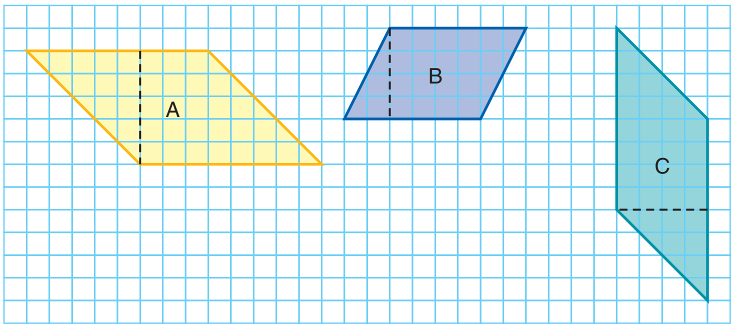


Explore

You will need scissors and 1-cm grid paper.

➤ Copy Parallelogram A on grid paper.
Estimate, then find, the area of the parallelogram.

➤ Cut out the parallelogram.
Then, cut along the broken line segment.



➤ Arrange the two pieces to form a rectangle.
What is the area of the rectangle?
How does the area of the rectangle compare to the area of the parallelogram?

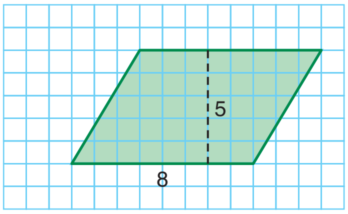
➤ Repeat the activity for Parallelograms B and C.

Reflect & Share

Share your work with another pair of classmates.
Can every parallelogram be changed into a rectangle by cutting and moving one piece? Explain.
Work together to write a rule for finding the area of a parallelogram.

Connect

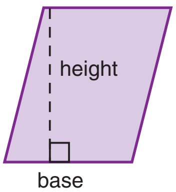
To estimate the area of this parallelogram, count the whole squares and the part squares that are one-half or greater.



There are:
- 33 whole squares
- 8 part squares that are one-half or greater

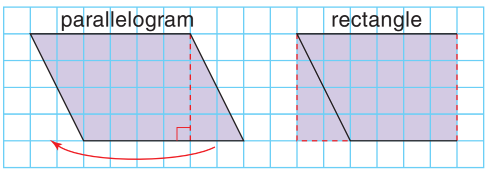
The area of this parallelogram is about 41 square units.

Any side of a parallelogram is a **base** of the parallelogram.
The **height** of a parallelogram is the length of a line segment that joins parallel sides and is perpendicular to the base.

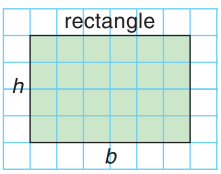


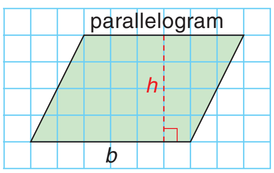
Recall that both a rectangle and a square are parallelograms.

Any parallelogram that is not a rectangle can be "cut" and rearranged to form a rectangle. Here is one way to do this.



The parallelogram and the rectangle have the same area.
The area of a parallelogram is equal to the area of a rectangle with the same height and base.
To find the area of a parallelogram, multiply the base by the height.





b represents the base.
h represents the height.

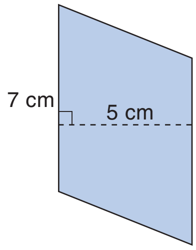
Area of rectangle:
$A = bh$

Area of parallelogram:
$A = bh$

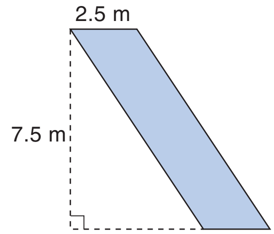
Example

Calculate the area of each parallelogram.

a)



b)



The height can be drawn outside the parallelogram.

A Solution

The area of a parallelogram is given by the formula $A = bh$.

a) $A = bh$

Substitute: $b = 7$ and $h = 5$

$A = 7 \times 5$

$= 35$

The area of the parallelogram is 35 cm^2.

b) $A = bh$

Substitute: $b = 2.5$ and $h = 7.5$

$A = 2.5 \times 7.5$

$= 18.75$

The area of the parallelogram is 18.75 m^2.

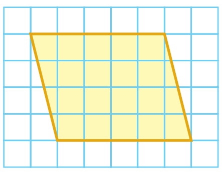
Practice

1. i) Copy each parallelogram on 1-cm grid paper.

ii) Show how the parallelogram can be rearranged to form a rectangle.

iii) Estimate, then find, the area of each parallelogram.

a)



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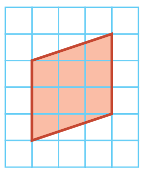
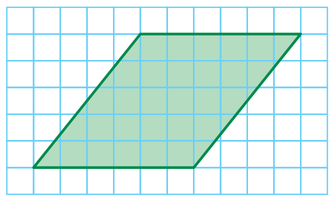
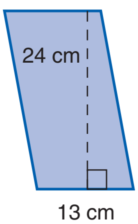


c)

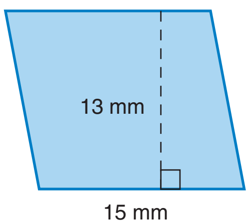


2. Find the area of each parallelogram.

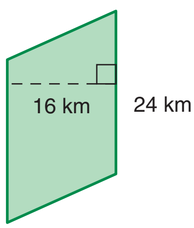
a)



b)



c)



3. a) On 1-cm grid paper, draw 3 different parallelograms with base 3 cm and height 7 cm.

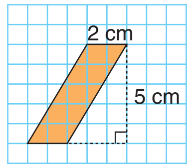
b) Find the area of each parallelogram you drew in part a. What do you notice?

4. Repeat question 3. This time, you choose the base and height.
Are your conclusions the same as in question 3? Why or why not?

5. Copy this parallelogram on 1-cm grid paper.

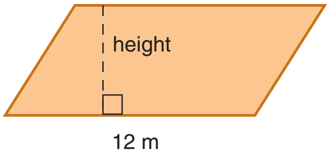
a) Show how this parallelogram could be rearranged to form a rectangle.

b) Find the area of the parallelogram.

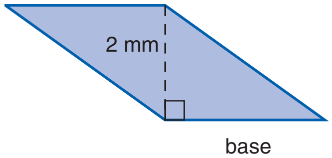


6. Use the given area to find the base or the height of each parallelogram.

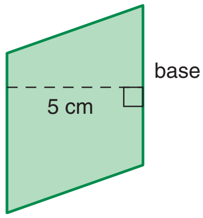
a) Area $= 60\ m^2$



b) Area $= 6\ mm^2$



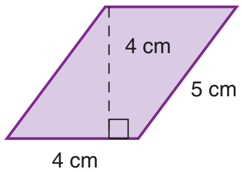
c) Area $= 30\ cm^2$



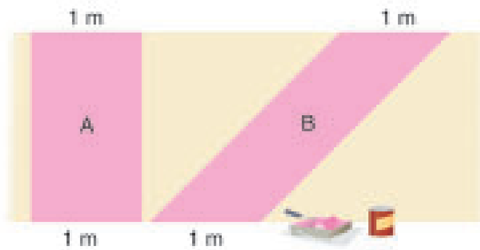
7. On 1-cm grid paper, draw as many different parallelograms as you can with each area.

a) $10\ cm^2$ **b)** $18\ cm^2$ **c)** $28\ cm^2$

8. A student says the area of this parallelogram is $20\ cm^2$.
Explain the student's error.



9. **Assessment Focus** Sasha is buying paint for a design on a wall. Here is part of the design.
Sasha says Shape B will need more paint than Shape A.
Do you agree? Why or why not?



10. **Take It Further** A restaurant owner built a patio in front of his store to attract more customers.

a) What is the area of the patio?

b) What is the total area of the patio and gardens?

c) How can you find the area of the gardens?

Show your work.

Reflect

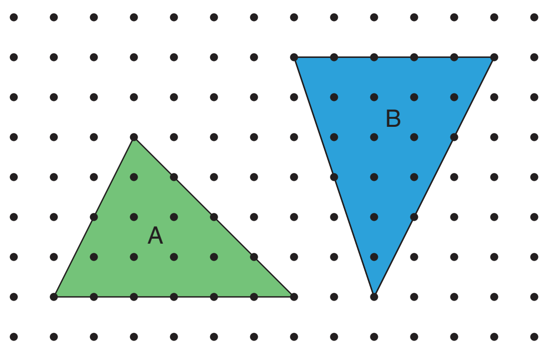
How can you use what you know about rectangles to help you find the area of a parallelogram?
Use an example to explain.

4.4 Area of a Triangle

Focus Develop and use a formula to find the area of a triangle.

Explore

You will need a geoboard, geobands, and dot paper.



➤ Make Triangle A on a geoboard.
Add a second geoband to Triangle A to make a parallelogram with the same base and height.
This is called a *related* parallelogram.
Make as many different parallelograms as you can.
How does the area of the parallelogram compare to the area of Triangle A each time?
Record your work on dot paper.

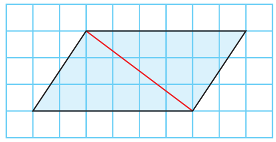
➤ Repeat the activity with Triangle B.

➤ What is the area of Triangle A? Triangle B?
What strategy did you use to find the areas?

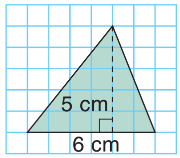
Reflect & Share

Share the different parallelograms you made with another pair of classmates.
Discuss the strategies you used to find the area of each triangle.
How did you use what you know about a parallelogram to find the area of a triangle?
Work together to write a rule for finding the area of a triangle.

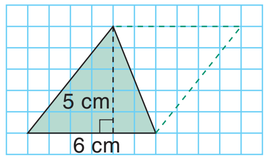
Connect



When we draw a diagonal in a parallelogram, we make two congruent triangles.
Congruent triangles have the same area.
The area of the two congruent triangles is equal to the area of the parallelogram that contains them.
So, the area of one triangle is $\frac{1}{2}$ the area of the parallelogram.



To find the area of this triangle:



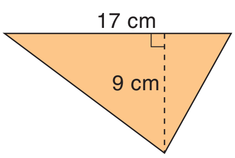
Complete a parallelogram on one side of the triangle.
The area of the parallelogram is:
$A = \text{base} \times \text{height}$, or $A = bh$
So, $A = 6 \times 5$
$\quad = 30$
The area of the parallelogram is 30 cm^2.
So, the area of the triangle is: $\frac{1}{2}$ of 30 cm^2 = 15 cm^2

We can write a formula for the area of a triangle.
The area of a parallelogram is:
$A = \text{base} \times \text{height}$
So, the area of a triangle is:
$A = \text{one-half of base} \times \text{height}$
$A = bh \div 2$, which can be written as $A = \frac{bh}{2}$

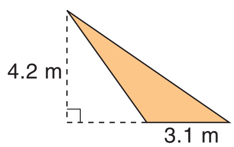
Example

Find the area of each triangle.

a)



b)



For an obtuse triangle, the height might be drawn outside the triangle.

A Solution

a) $A = \frac{bh}{2}$

Substitute: $b = 17$ and $h = 9$

$A = \frac{17 \times 9}{2}$

$= \frac{153}{2}$

$= 76.5$

The area is 76.5 cm^2.

b) $A = \frac{bh}{2}$

Substitute: $b = 3.1$ and $h = 4.2$

$A = \frac{3.1 \times 4.2}{2}$

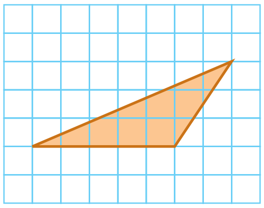
$= \frac{13.02}{2}$

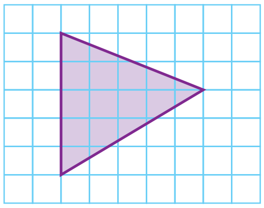
$= 6.51$

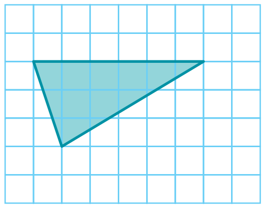
The area is 6.51 m^2.

Practice

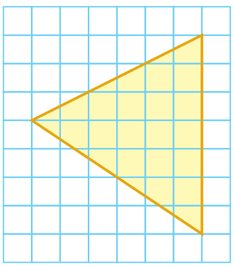
1. Copy each triangle on 1-cm grid paper. Draw a related parallelogram.

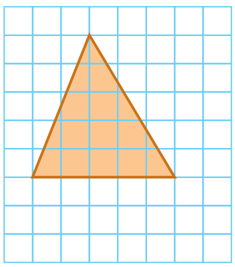
a) 

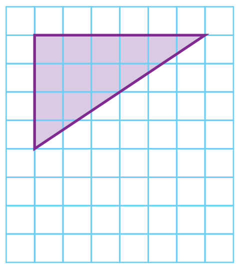
b) 

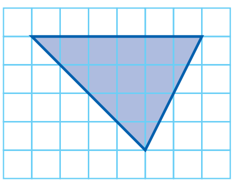
c) 

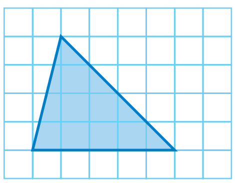
2. Each triangle is drawn on 1-cm grid paper.
Find the area of each triangle. Use a geoboard if you can.

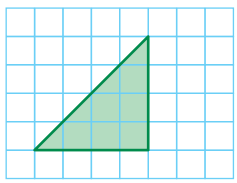
a) 

b) 

c) 

d) 

e) 

f) 

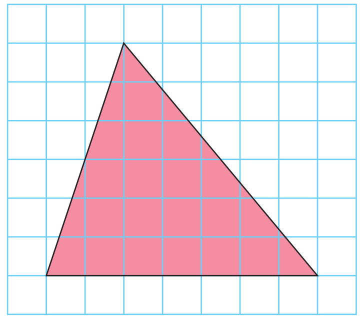
3. Draw two right triangles on 1-cm grid paper.

a) Record the base and the height of each triangle.

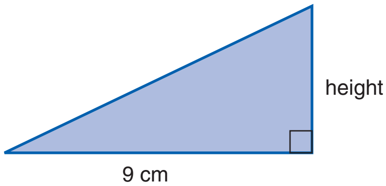
b) What do you notice about the height of a right triangle?

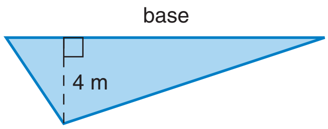
c) Find the area of each triangle you drew.

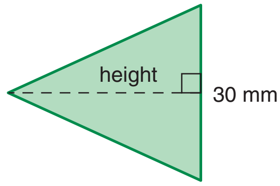
4. a) Find the area of this triangle.
   b) Use 1-cm grid paper.
      How many different parallelograms can you draw that have the same base and the same height as this triangle?
      Sketch each parallelogram.
   c) Find the area of each parallelogram.
      What do you notice?



5. Use the given area to find the base or height of each triangle.
   How could you check your answers?
   a) Area = 18 cm^2
   b) Area = 32 m^2
   c) Area = 480 mm^2







6. Use 1-cm grid paper.
   a) Draw 3 different triangles with each base and height.
      i) base: 1 cm; height: 12 cm
      ii) base: 2 cm; height: 6 cm
      iii) base: 3 cm; height: 4 cm
   b) Find the area of each triangle you drew in part a.
      What do you notice?

7. On 1-cm grid paper, draw two different triangles with each area below.
   Label the base and height each time.
   How do you know these measures are correct?
   a) 14 cm^2
   b) 10 cm^2
   c) 8 cm^2

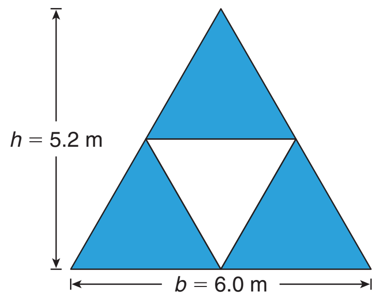
8. a) Draw any triangle on grid paper.
      What happens to the area of the triangle in each case?
      i) the base is doubled
      ii) both the height and the base are doubled
      iii) both the height and the base are tripled
   b) What could you do to the triangle you drew in part a to triple its area?
      Explain why this would triple the area.

9. **Assessment Focus**

This triangle is made from 4 congruent triangles.
Three triangles are to be painted blue.
The fourth triangle is not to be painted.

a) What is the area that is to be painted?
Show your work.

b) The paint is sold in 1-L cans.
One litre of paint covers 5.5 m^2.
How many cans of paint are needed?
What assumptions did you make?



The height is approximate.

10. Look at the diagram to the right.

a) How many triangles do you see?

b) How are the triangles related?

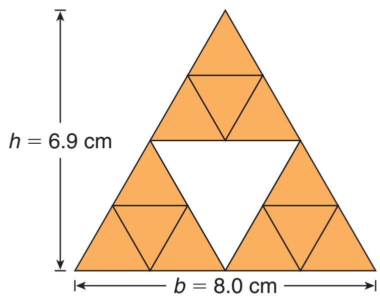
c) How many parallelograms do you see?

d) Find the area of the large triangle.

e) Find the area of one medium-sized triangle.

f) Find the area of one small triangle.

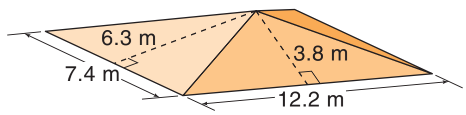
g) Find the area of a parallelogram of your choice.



The height is approximate.

11. **Take It Further**

A local park has a pavilion to provide shelter.
The pavilion has a roof the shape of a rectangular pyramid.



a) What is the total area of all four parts of the roof?

b) One sheet of plywood is 240 cm by 120 cm.
What is the least number of sheets of plywood needed to cover the roof?
Explain how you got your answer.

Reflect

What do you know about finding the area of a triangle?

4.5 Area of a Circle

Focus Develop and use a formula to find the area of a circle.

Explore

You will need one set of fraction circles, masking tape, and a ruler.

➤ Each of you chooses one circle from the set of fraction circles. The circle you choose should have an even number of sectors, and at least 4 sectors.

➤ Each of you cuts 3 strips of masking tape:
- 2 short strips
- 1 strip at least 15 cm long

Use the short strips to fasten the long strip face up on the table.

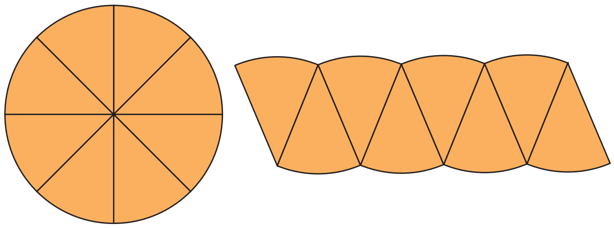
➤ Arrange all your circle sectors on the tape to approximate a parallelogram.
Trace your parallelogram, then use a ruler to make the horizontal sides straight.
Calculate the area of the parallelogram. Estimate the area of the circle.
How does the area of the parallelogram compare to the area of the circle?

Reflect & Share

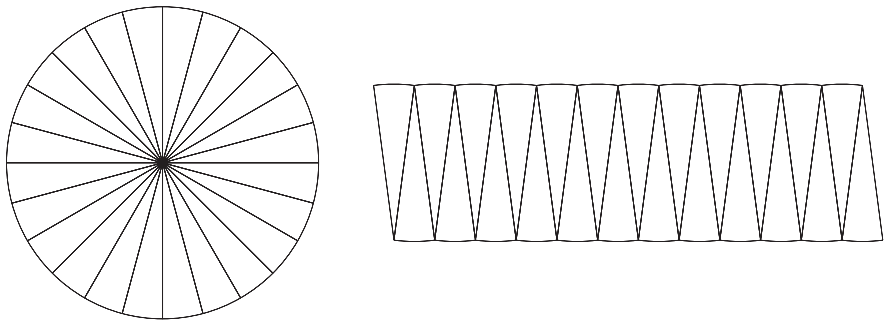
Compare your measure of the area of the circle with the measures of your group members.
Which area do you think is closest to the area of the circle? Why?
How could you improve your estimate for the area?
Which circle measure best represents the height of the parallelogram?
The base? Work together to write a formula for the area of a circle.

Connect

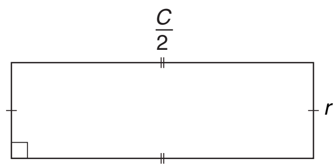
Suppose a circle was cut into 8 congruent sectors.
The 8 sectors were then arranged to approximate a parallelogram.



The more congruent sectors we use, the closer the area of the parallelogram is to the area of the circle.
Here is a circle cut into 24 congruent sectors.
The 24 sectors were then arranged to approximate a parallelogram.



The greater the number of sectors, the more the shape looks like a rectangle.
The sum of the two longer sides of the rectangle is equal to the circumference, C.
So, each longer side, or the base of the rectangle, is one-half the circumference of the circle, or $\frac{C}{2}$.

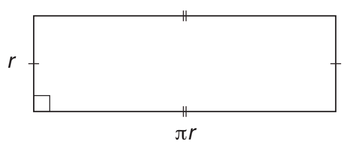


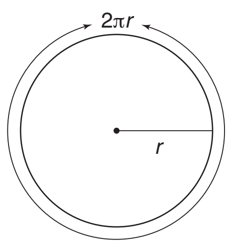
But $C = 2\pi r$

So, the base of the rectangle $= \frac{2\pi r}{2}$

$= \pi r$

Each of the two shorter sides is equal to the radius, r.



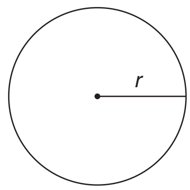


The area of a rectangle is: base $\times$ height
The base is πr. The height is r.
So, the area of the rectangle is: $\pi r \times r = \pi r^2$

When a number or variable is multiplied by itself we write: $7 \times 7 = 7^2$
$r \times r = r^2$

Since the rectangle is made from all sectors of the circle, the rectangle and the circle have the same area.
So, the area, A, of the circle with radius r is $A = \pi r^2$.

We can use this formula to find the area of any circle when we know its radius.



$A = \pi r^2$

Example

The face of a dime has diameter 1.8 cm.

a) Calculate the area.
Give the answer to two decimal places.

b) Estimate to check the answer is reasonable.

A Solution

The diameter of the face of a dime is 1.8 cm.
So, its radius is: $\frac{1.8\text{ cm}}{2} = 0.9$ cm

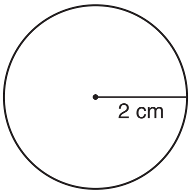
a) Use the formula: $A = \pi r^2$
Substitute: $r = 0.9$
$A = \pi \times 0.9^2$
Use a calculator.
$A \doteq 2.544\ 69$
The area of the face of the dime is 2.54 cm^2 to two decimal places.

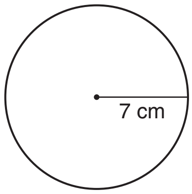
If your calculator does not have an x^2 key, key in 0.9×0.9 instead.

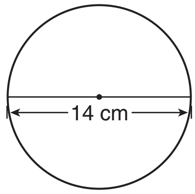
b) Recall that $\pi \doteq 3$.
So, the area of the face of the dime is about $3r^2$.
$r \doteq 1$
So, $r^2 = 1$
and $3r^2 = 3 \times 1$
$= 3$
The area of the face of the dime is approximately 3 cm^2.
Since the calculated area, 2.54 cm^2, is close to 3 cm^2, the answer is reasonable.

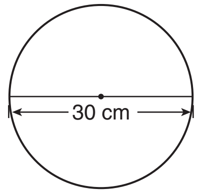
Practice

1. Calculate the area of each circle.
Estimate to check your answers are reasonable.

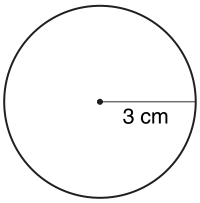
a) 

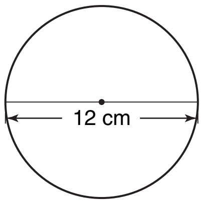
b) 

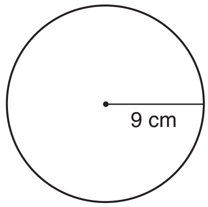
c) 

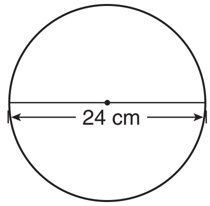
d) 

2. Calculate the area of each circle. Give your answers to two decimal places.
Estimate to check your answers are reasonable.

a) 

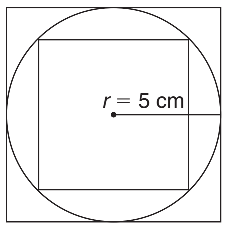
b) 

c) 

d) 

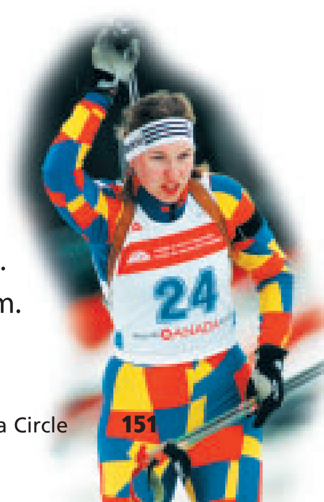
3. Use the results of questions 1 and 2. What happens to the area in each case?
a) You double the radius of a circle.
b) You triple the radius of a circle.
c) You quadruple the radius of a circle.
Justify your answers.

4. **Assessment Focus** Use 1-cm grid paper.
Draw a circle with radius 5 cm.
Draw a square outside the circle that just encloses the circle.
Draw a square inside the circle
so that its vertices lie on the circle.
Measure the sides of the squares.



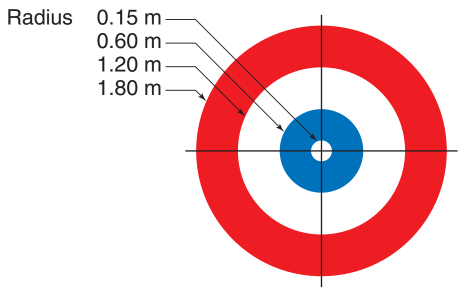
a) How can you use the areas of the two squares to estimate the area of the circle?
b) Check your estimate in part a by calculating the area of the circle.
c) Repeat the activity for circles with different radii.
Record your results. Show your work.

5. In the biathlon, athletes shoot at targets. Find the area of each target.
a) The target for the athlete who is standing is a circle with diameter 11.5 cm.
b) The target for the athlete who is lying down is a circle with diameter 4.5 cm.
Give the answers to the nearest square centimetre.

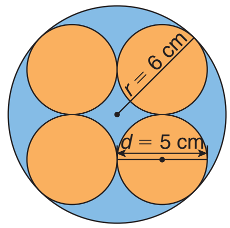


6. In curling, the target area is a bull's eye with 4 concentric circles.
   a) Calculate the area of the smallest circle.
   b) When a smaller circle overlaps a larger circle, a ring is formed.
      Calculate the area of each ring on the target area.
   Give your answers to 4 decimal places.

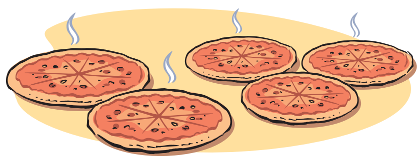
Concentric circles have the same centre.



7. **Take It Further**
   A circle with radius 6 cm contains 4 small circles.
   Each small circle has diameter 5 cm.
   Each small circle touches two other small circles and the large circle.
   a) Find the area of the large circle.
   b) Find the area of one small circle.
   c) Find the area of the region that is shaded yellow.



8. **Take It Further** A large pizza has diameter 35 cm.
   Two large pizzas cost $19.99.
   A medium pizza has diameter 30 cm.
   Three medium pizzas cost $24.99.
   Which is the better deal: 2 large pizzas or 3 medium pizzas?
   Justify your answer.



Math Link

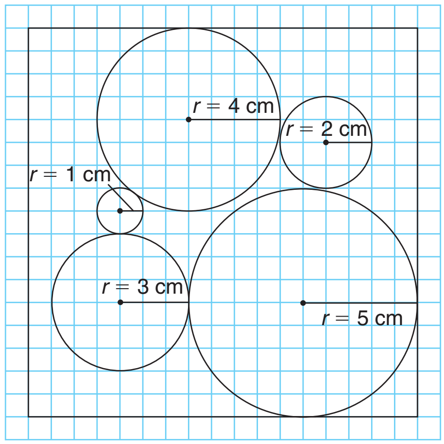
Agriculture: Crop Circles
In Red Deer, Alberta, on September 17, 2001, a crop circle formation was discovered that contained 7 circles. The circle shown has diameter about 10 m. This circle destroyed some wheat crop. What area of wheat crop was lost in this crop circle?

Reflect

You have learned two formulas for measurements of a circle.
How do you remember which formula to use for the area of a circle?

Packing Circles

These circles are packed in a square.
In this game, you will pack circles in other shapes.



YOU WILL NEED

2 sheets of circles
scissors
ruler
compass
calculator

NUMBER OF PLAYERS

2

GOAL OF THE GAME

Construct the circle, triangle, and parallelogram with the lesser area.

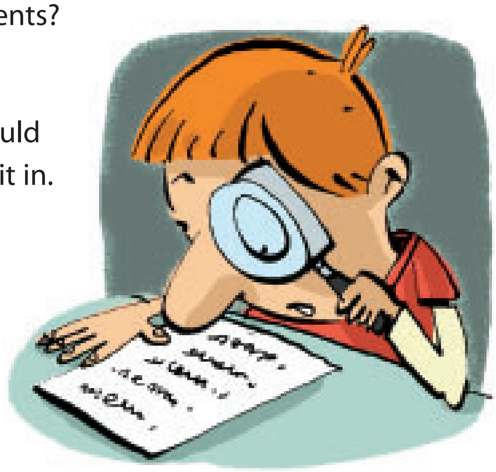
What strategies did you use to pack your circles to construct the shape with the lesser area?

HOW TO PLAY THE GAME:

1. Each player cuts out one sheet of circles.
2. Each player arranges his 5 circles so they are packed tightly together.
3. Use a compass. Draw a circle that encloses these circles.
4. Find the area of the enclosing circle.
   The player whose circle has the lesser area scores 2 points.
5. Pack the circles again.
   This time draw the parallelogram that encloses the circles.
   Find the area of the parallelogram.
   The player whose parallelogram has the lesser area scores 2 points.
6. Repeat *Step 5*. This time use a triangle to enclose the circles.
7. The player with the higher score wins.

Reading for Accuracy—Checking Your Work

Do you make careless mistakes on homework assignments? On quizzes or tests?



To help improve your accuracy and test scores, you should get in the habit of checking your work before handing it in.

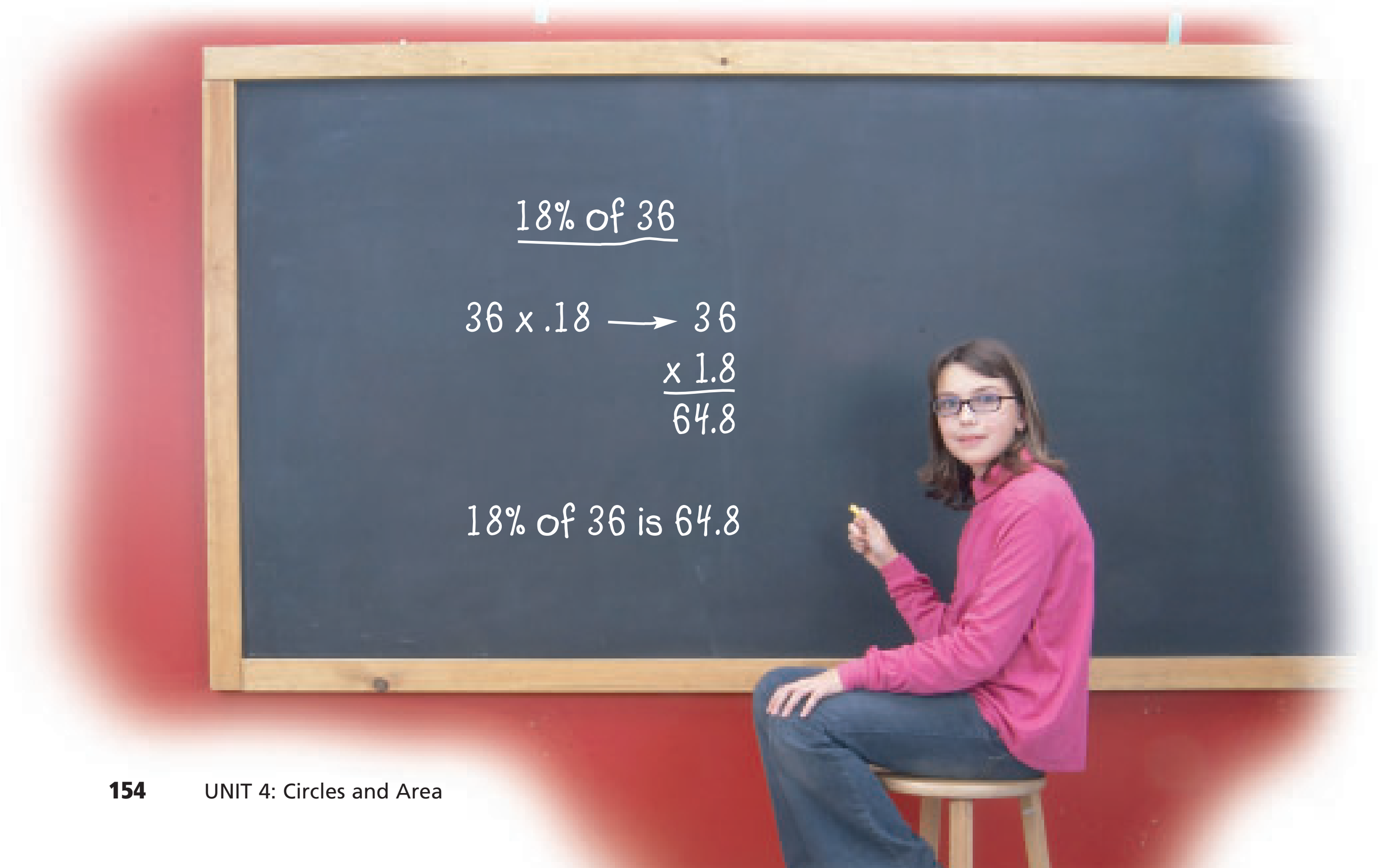
The three most common types of mistakes are:
- copying errors
- notation errors
- calculation errors

Copying Errors

A copying error occurs when you copy the question or numbers in the question incorrectly.

Find the copying error in this solution.
How can you tell that the solution is not reasonable? Explain.

1. Find 18% of 36.



Notation Errors

A notation error occurs when you use a math symbol incorrectly.

Find the notation error in this solution.

2. Evaluate: (–8) + (+3) – (+2)

Evaluate (-8) + (+3) - (+2)

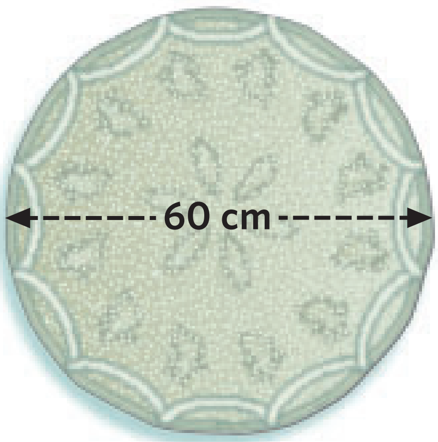
(-8) + (+3) - (+2) = (-5) - (+2)
= (+5) + (+2)
= (+7)

Calculation Errors

A calculation error occurs when you make a mistake in your calculations.

Find the calculation error in this solution.
How can you tell that the solution is not reasonable? Explain.

3. A circular mat has diameter 60 cm.
What is the area of the mat?



The diameter of the mat is 60 cm.
So, its radius is: 60 cm/2 = 30 cm
Use the formula $A = \pi r^2$
Substitute: r = 30
$A = \pi \times 30^2$
$= \pi \times 9000$
$\doteq 28\ 274$
The area of the mat is about 28 274 cm^2.

4. Correct the error you found in each solution to find the correct answer.
Show your work.

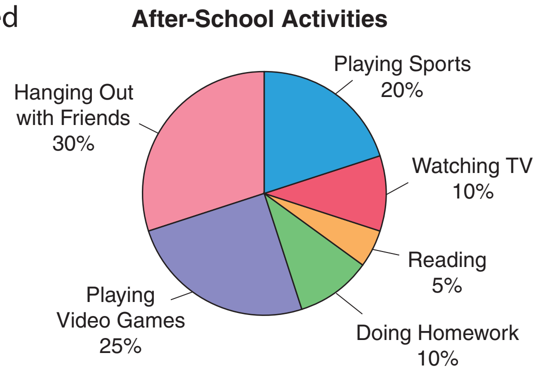
4.6 Interpreting Circle Graphs

Focus Interpret circle graphs to solve problems.

We can apply what we have learned about circles to interpret a new type of graph.

Explore

Sixty Grade 7 students at l'école Orléans were surveyed to find out their favourite after-school activity. The results are shown on the circle graph.



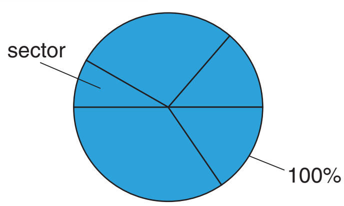
Which activity is most popular? Least popular? How do you know this from looking at the graph? How many students prefer each type of after-school activity? Which activity is the favourite for about $\frac{1}{3}$ of the students? Why do you think so? Write 3 more things you know from looking at the graph.

Reflect & Share

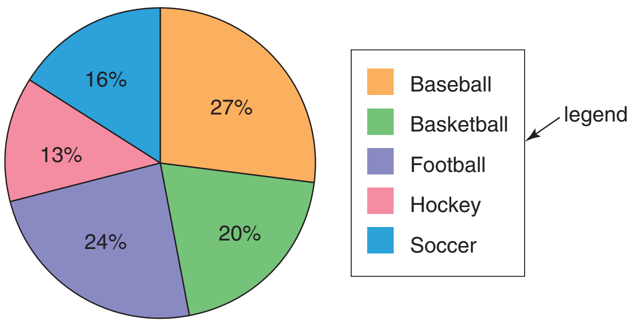
Compare your answers with those of another pair of classmates. What do you notice about the sum of the percents? Explain.

Connect

In a **circle graph**, data are shown as parts of one whole. Each **sector** of a circle graph represents a percent of the whole circle. The whole circle represents 100%.



A circle graph has a title.
Each sector is labelled with a category and a percent.
A circle graph compares the number in each category to the total number.
That is, a fraction of the circle represents the same fraction of the total.
Sometimes, a circle graph has a **legend** that shows what category each sector represents.
In this case, only the percents are shown on the graph.

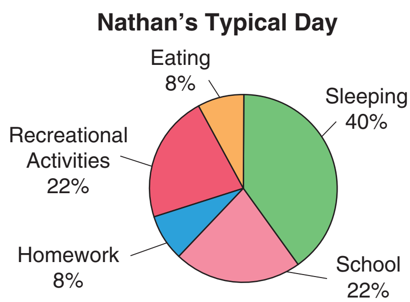


Example

This graph shows Nathan's typical day.

a) Which activity does Nathan do about $\frac{1}{4}$ of the time?

b) About how many hours does Nathan spend on each activity?
Check that the answers are reasonable.



A Solution

a) Each of the sectors for "School" and "Recreational Activities" is about $\frac{1}{4}$ of the graph. 22% is close to 25%, which is $\frac{1}{4}$.
So, Nathan is in school about $\frac{1}{4}$ of the day.
He also participates in recreational activities about $\frac{1}{4}$ of the day.

b) ➤ From the circle graph, Nathan spends 40% of his day sleeping.
There are 24 h in a day.
Find 40% of 24.
$40\% = \frac{40}{100} = 0.4$
Multiply: $0.4 \times 24 = 9.6$
Nathan spends about 10 h sleeping.

9.6 is closer to 10 than to 9.

➤ Nathan spends 22% of his day in school.
Find 22% of 24.
$22\% = \frac{22}{100} = 0.22$
Multiply: $0.22 \times 24 = 5.28$
Nathan spends about 5 h in school.
Nathan also spends about 5 h doing recreational activities.

5.28 is closer to 5 than to 6.

➤ Nathan spends 8% of his day doing homework.
Find 8% of 24.
$8\% = \frac{8}{100} = 0.08$
Multiply: 0.08×24
Multiply as you would whole numbers.

$$\begin{array}{r} 24 \\ \times\ 8 \\ \hline 192 \end{array}$$

Estimate to place the decimal point.
$0.1 \times 24 = 2.4$
So, $0.08 \times 24 = 1.92$
Nathan spends about 2 h doing homework.
Nathan also spends about 2 h eating.

1.92 is closer to 2 than to 1.

The total number of hours spent on all activities should be 24, the number of hours in a day:
$9.6 + 5.28 + 5.28 + 1.92 + 1.92 = 24$
So, the answers are reasonable.

Add the exact times, *not* the approximate times.

Practice

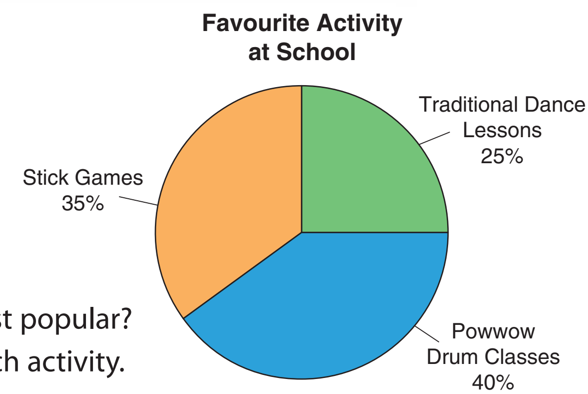
1. This circle graph shows the most popular activities in a First Nations school.
There are 500 students in the school.
All students voted.

a) Which activity did about $\frac{1}{4}$ of the students choose?
How can you tell by looking at the graph?

b) Which activity is the most popular? The least popular?

c) Find the number of students who chose each activity.

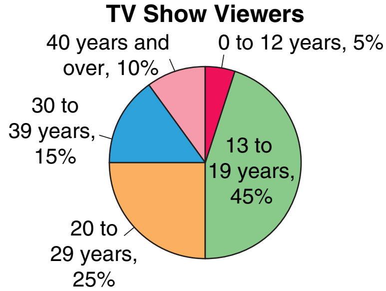
d) How can you check your answers to part c?



2. This circle graph shows the ages of viewers of a TV show.
One week, approximately 250 000 viewers tuned in.

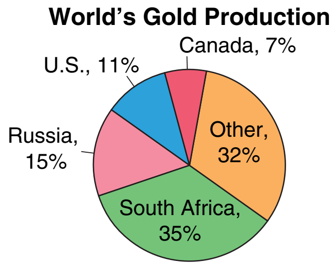
a) Which two age groups together make up $\frac{1}{2}$ of the viewers?

b) How many viewers were in each age group?
   i) 13 to 19 ii) 20 to 29 iii) 40 and over



3. This graph shows the world's gold production for a particular year.
In this year, the world's gold production was approximately 2300 t.
About how much gold would have been produced in each country?

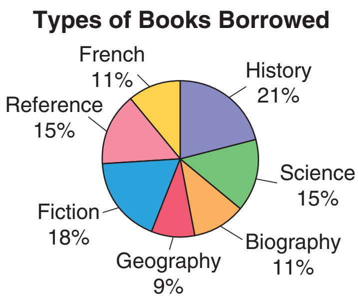
a) Canada b) South Africa



4. The school library budget to buy new books is $5000.
The librarian has this circle graph to show the types of books students borrowed in one year.

a) How much money should be spent on each type of book? How do you know?

b) Explain how you can check your answers in part a.



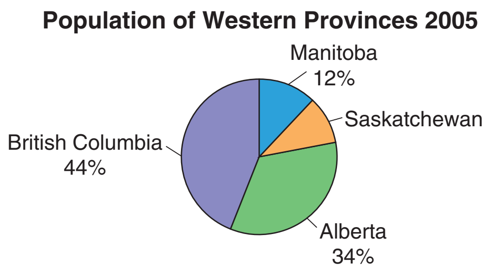
5. **Assessment Focus** This circle graph shows the populations of the 4 Western Canadian provinces in 2005.
The percent for Saskatchewan is not shown.

a) What percent of the population lived in Saskatchewan? How do you know?

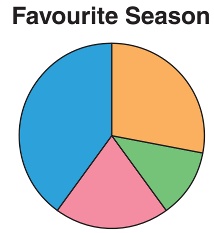
b) List the provinces in order from least to greatest population.
   How did the circle graph help you do this?

c) In 2005, the total population of the Western provinces was about 9 683 000 people.
   Calculate the population of each province, to the nearest thousand.

d) What else do you know from looking at the circle graph? Write as much as you can.



6. Gaston collected data about the favourite season of his classmates.



Classmates' Favourite Season

Season	Autumn	Winter	Spring	Summer
Number of Students	7	3	5	10

He recorded the results in a circle graph.
The graph is not complete.

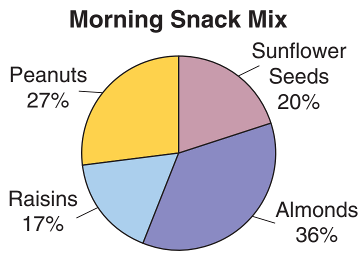
a) How many students were surveyed?

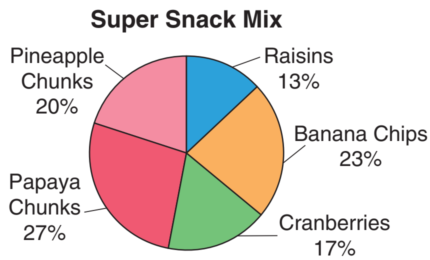
b) Write the number of students who chose each season as a fraction of the total number of students, then as a percent.

c) Explain how you can check your answers to part b.

d) Sketch the graph. Label each sector with its name and percent. How did you do this?

7. These circle graphs show the percent of ingredients in two 150-g samples of different snack mixes.





a) For each snack mix, calculate the mass, in grams, of each ingredient.

b) About what mass of raisins would you expect to find in a 300-g sample of each mix? What assumptions did you make?

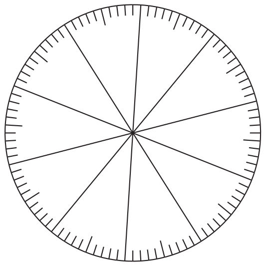
Reflect

Search newspapers, magazines, and the Internet to find examples of circle graphs.
Cut out or print the graphs.
How are they the same? How are they different?
Why were circle graphs used to display these data?

4.7 Drawing Circle Graphs

Focus Construct circle graphs to display data.

This is a **percent circle**.
The circle is divided into 100 congruent parts.
Each part is 1% of the whole circle.
You can draw a circle graph on a percent circle.



Explore

Your teacher will give you a percent circle.
Students in a Grade 7 class were asked how many siblings they have.
Here are the results.

0 Siblings	1 Sibling	2 Siblings	More than 2 Siblings
3	13	8	1

Write each number of students as a fraction of the total number.
Then write the fraction as a percent.

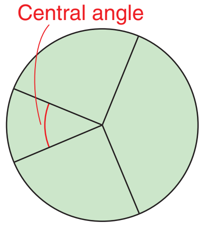
Use the percent circle.
Draw a circle graph to display the data.
Write 2 questions you can answer by looking at the graph.

Reflect & Share

Trade questions with another pair of classmates.
Use your graph to answer your classmates' questions.
Compare graphs. If they are different, try to find out why.
How did you use fractions and percents to draw a circle graph?

Connect

Recall that a circle graph shows how parts of a set of data compare with the whole set.
Each piece of data is written as a fraction of the whole.
Each fraction is then written as a percent.
Sectors of a percent circle are coloured to represent these percents.
The sum of the **central angles** is 360°.
A central angle is also called a **sector angle**.



Example

All the students in two Grade 7 classes were asked how they get to school each day. Here are the results: 9 rode their bikes, 11 walked, 17 rode the bus, and 13 were driven by car. Construct a circle graph to illustrate these data.

A Solution

➤ For each type of transport:
Write the number of students as a fraction of 50, the total number of students.
Then write each fraction as a decimal and as a percent.

Bike: $\frac{9}{50} = \frac{18}{100} = 0.18 = 18\%$ Walk: $\frac{11}{50} = \frac{22}{100} = 0.22 = 22\%$

Bus: $\frac{17}{50} = \frac{34}{100} = 0.34 = 34\%$ Car: $\frac{13}{50} = \frac{26}{100} = 0.26 = 26\%$

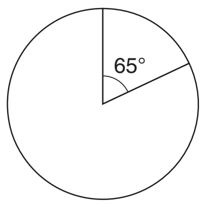
The circle represents all the types of transport.
To check, add the percents.
The sum should be 100%.
$18\% + 22\% + 34\% + 26\% = 100\%$

Another Strategy
We could use a percent circle to graph these data.

➤ To find the sector angle for each type of transport, multiply each decimal by 360°.
Write each angle to the nearest degree, when necessary.
Bike 18%: $0.18 \times 360° = 64.8° \doteq 65°$
Walk 22%: $0.22 \times 360° = 79.2° \doteq 79°$
Bus 34%: $0.34 \times 360° = 122.4° \doteq 122°$
Car 26%: $0.26 \times 360° = 93.6° \doteq 94°$

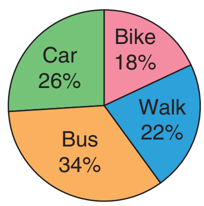
Check:
$64.8° + 79.2° + 122.4° + 93.6° = 360°$

➤ Construct a circle.
Use a protractor to construct each sector angle.
Start with the smallest angle.
Draw a radius. Measure 65°.



Start the next sector where the previous sector finished.
Label each sector with its name and percent.
Write a title for the graph.

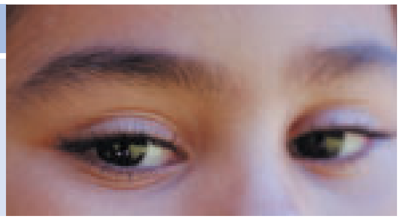
How Students Get to School



Practice

1. The table shows the number of Grade 7 students with each eye colour at Northern Public School.

Eye Colour	Number of Students
Blue	12
Brown	24
Green	8
Grey	6



a) Find the total number of students.
b) Write the number of students with each eye colour as a fraction of the total number of students.
c) Write each fraction as a percent.
d) Draw a circle graph to represent these data.

2. In a telephone survey, 400 people voted for their favourite radio station.

Radio Station	Votes
MAJIC99	88
EASY2	?
ROCK1	120
HITS2	100

a) How many people chose EASY2?
b) Write the number of people who voted for each station as a fraction of the total number who voted.
Then write each fraction as a percent.
c) Draw a circle graph to display the results of the survey.

3. **Assessment Focus** This table shows the method of transport used by U.S. residents entering Canada in one year.

United States Residents Entering Canada

Method of Transport	Number
Automobile	32 000 000
Plane	4 000 000
Train	400 000
Bus	1 600 000
Boat	1 200 000
Other	800 000

a) How many U.S. residents visited Canada that year?
b) What fraction of U.S. residents entered Canada by boat?
c) What percent of U.S. residents entered Canada by plane?
d) Display the data in a circle graph.
e) What else do you know from the table or circle graph?
Write as much as you can.

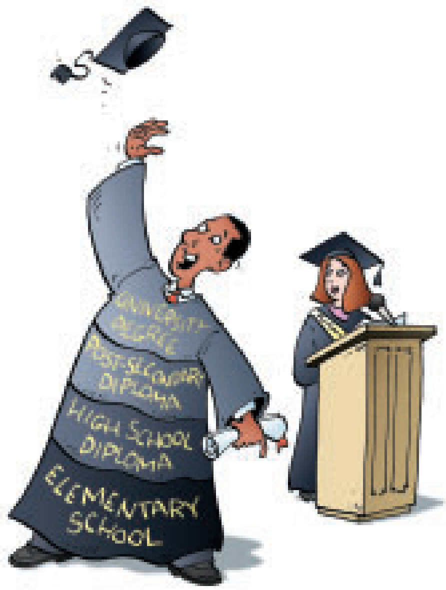
4. Can the data in each table below be displayed in a circle graph? Explain.

a)

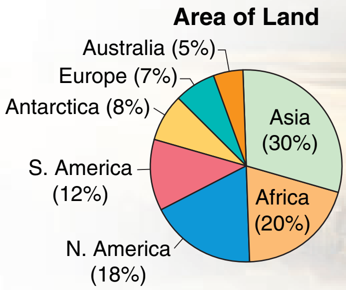
Educational Attainment of Canadians	
0 to 8 years of elementary school	10%
Some secondary school	17%
Graduated from high school	20%
Some post-secondary education	9%
Post-secondary certificate or diploma	28%
University degree	16%

b)

Canadian Households with These Conveniences	
Automobile	64%
Cell phone	42%
Dishwasher	51%
Internet	42%



5. **Take It Further** This circle graph shows the percent of land occupied by each continent. The area of North America is approximately 220 million km^2. Use the percents in the circle graph. Find the approximate area of each of the other continents, to the nearest million square kilometres.



Reflect

When is it most appropriate to show data using a circle graph?
When is it not appropriate?

Using a Spreadsheet to Create Circle Graphs

Focus Display data on a circle graph using spreadsheets.

Spreadsheet software can be used to record, then graph, data.
This table shows how Stacy budgets her money each month.

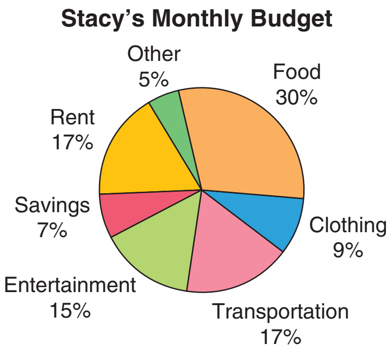
Stacy's Monthly Budget

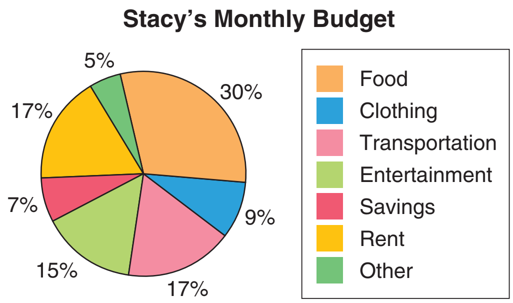
Category	Amount ($)
Food	160
Clothing	47
Transportation	92
Entertainment	78
Savings	35
Rent	87
Other	28

Enter the data into rows and columns of a spreadsheet.
Highlight the data. Do not include the column heads.

	A	B
1	Category	Amount ($)
2	Food	160
3	Clothing	47
4	Transportation	92
5	Entertainment	78
6	Savings	35
7	Rent	87
8	Other	28

Click the graph/chart icon. In most spreadsheet programs, circle graphs are called **pie charts**. Select *pie chart*.
Investigate different ways of labelling the graph.
Your graph should look similar to one of the graphs on the following page.





This circle graph shows a legend at the right. The legend shows what category each sector represents.

These data are from *Statistics Canada*.

1. **a)** Use a spreadsheet.
Create a circle graph to display these data.
b) Write 3 questions about your graph.
Answer your questions.
c) Compare your questions with those of a classmate.
What else do you know from the table or the graph?

Population by Province and Territory, October 2005

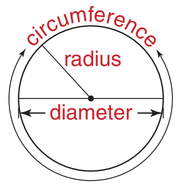
Region	Population
Newfoundland and Labrador	515 591
Prince Edward Island	138 278
Nova Scotia	938 116
New Brunswick	751 726
Quebec	7 616 645
Ontario	12 589 823
Manitoba	1 178 109
Saskatchewan	992 995
Alberta	3 281 296
British Columbia	4 271 210
Yukon Territories	31 235
Northwest Territories	42 965
Nunavut	30 133

Unit Review

What Do I Need to Know?

☑ Measurements in a Circle

The distance from the centre to a point on the circle is the *radius*. The distance across the circle, through the centre, is the *diameter*.
The distance around the circle is the *circumference*.



☑ Circle Relationships

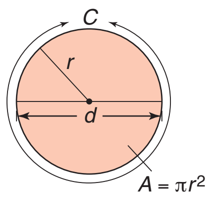
In a circle, let the radius be r, the diameter d, the circumference C, and the area A.

Then, $d = 2r$

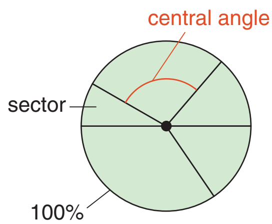
$\frac{d}{2} = r$

$C = 2\pi r$, or $C = \pi d$

$A = \pi r^2$

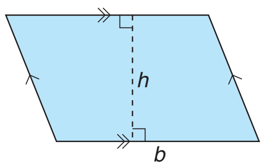


π is an irrational number that is approximately 3.14.
The sum of the central angles of a circle is 360°.

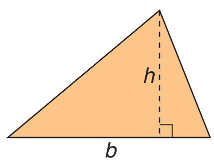


☑ Area Formulas

Parallelogram: $A = bh$
where b is the base and
h is the height



Triangle: $A = \frac{bh}{2}$ or
$A = bh \div 2$
where b is the base and h is the height



☑ Circle Graphs

In a circle graph, data are shown as parts of one whole.
The data are reported as a percent of the total, and the sum of the percents is 100%. The sum of the sector angles is 360°.

What Should I Be Able to Do?

LESSON

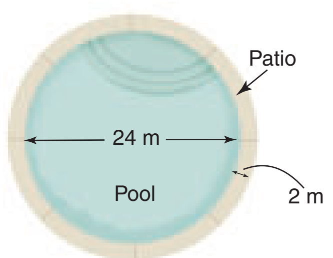
4.1 **1.** Draw a large circle without using a compass. Explain how to find the radius and diameter of the circle you have drawn.

2. Find the radius of a circle with each diameter.
a) 12 cm **b)** 20 cm **c)** 7 cm

3. Find the diameter of a circle with each radius.
a) 15 cm **b)** 22 cm **c)** 4.2 cm

4.2 **4.** The circumference of a large crater is about 219 m. What is the radius of the crater?

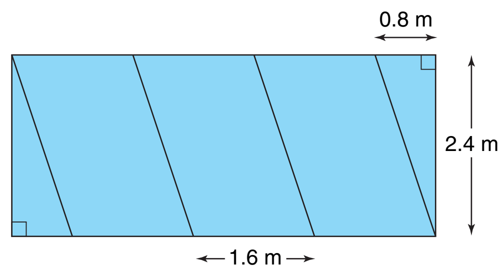
5. A circular pool has a circular concrete patio around it.
a) What is the circumference of the pool?
b) What is the combined radius of the pool and patio?
c) What is the circumference of the outside edge of the patio?



6. Mitra and Mel have different MP3 players. The circular control dial on each player is a different size. Calculate the circumference of the dial on each MP3 player.
a) Mitra's dial: diameter 30 mm
b) Mel's dial: radius 21 mm
c) Whose dial has the greater circumference? Explain.

4.3 **7.** On 0.5-cm grid paper, draw 3 different parallelograms with area 24 cm^2. What is the base and height of each parallelogram?

4.3 4.4 **8. a)** The window below consists of 5 pieces of glass. Each piece that is a parallelogram has base 1.6 m. What is the area of one parallelogram?



b) The base of each triangle in the window above is 0.8 m.
i) What is the area of one triangle?
ii) What is the area of the window? Explain how you found the area.

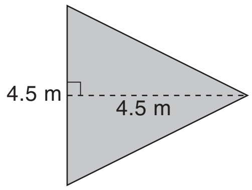
LESSON

9. On 0.5-cm grid paper, draw 3 different triangles with area 12 cm^2.

a) What is the base and height of each triangle?

b) How are the triangles related to the parallelograms in question 7?

10. Po Ling is planning to pour a concrete patio beside her house. It has the shape of a triangle. The contractor charges $125.00 for each square metre of patio.



What will the contractor charge for the patio?

4.5 **11.** A goat is tied to an 8-m rope in a field.

a) What area of the field can the goat graze?

b) What is the circumference of the area in part a?

12. Choose a radius. Draw a circle. Suppose you divide the radius by 2.

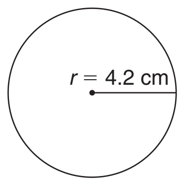
a) What happens to the circumference?

b) Explain what happens to the area.

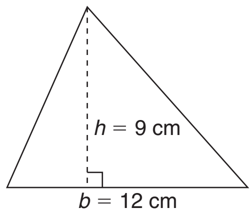
13. The diameter of a circular mirror is 28.5 cm. What is the area of the mirror? Give the answer to two decimal places.

14. Suppose you were to paint inside each shape below. Which shape would require the most paint? How did you find out?

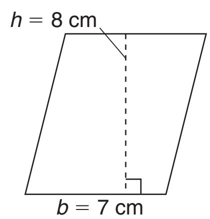
a)



b)



c)



LESSON

4.6

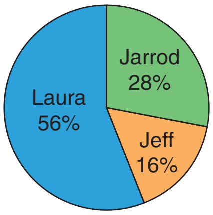
15. The results of the student council election are displayed on a circle graph. Five hundred students voted. The student with the most votes was named president.

a) Which student was named president? How do you know?

b) How many votes did each candidate receive?

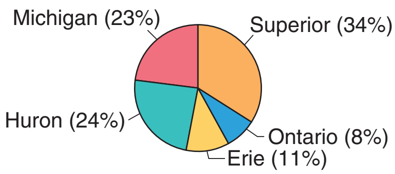
c) Write 2 other things you know from the graph.

Student Council Election Results



16. This circle graph shows the surface areas of the Great Lakes.

Areas of the Great Lakes



a) Which lake has a surface area about $\frac{1}{4}$ of the total area?

b) Explain why Lake Superior has that name.

c) The total area of the Great Lakes is about 244 000 km^2. Find the surface area of Lake Erie.

4.7

17. This table shows the approximate chemical and mineral composition of the human body.

Component	Percent
Water	62
Protein	17
Fat	15
Nitrogen	3
Calcium	2
Other	1

a) Draw a circle graph to display these data.

b) Jensen has mass 60 kg. About how many kilograms of Jensen's mass is water?

18. Here are the top 10 point scorers on the 2006 Canadian Women's Olympic Hockey Team. The table shows each player's province of birth.

Manitoba	Saskatchewan
Botterill	Wickenheiser
Quebec	**Ontario**
Ouellette Goyette Vaillancourt	Apps Campbell Hefford Piper Weatherston

a) What percent was born in each province?

b) Draw a circle graph to display the data in part a.

c) Why do you think more of these players come from Ontario than from any other province?

Practice Test

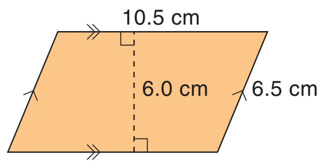
1. Draw a circle. Measure its radius.
   Calculate its diameter, circumference, and area.

2. The circular frame of this dream catcher has diameter 10 cm.
   a) How much wire is needed to make the outside frame?
   b) What is the area enclosed by the frame of this dream catcher?

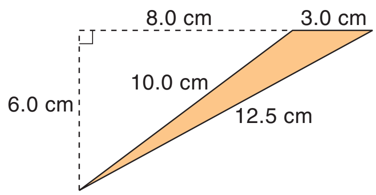
3. A circle is divided into 8 sectors.
   What is the sum of the central angles of the circle? Justify your answer.

4. Find the area of each shape. Explain your strategy.

   a)



   b)



5. a) How many different triangles and parallelograms can you sketch with area 20 cm^2?
   Have you sketched all possible shapes? Explain.
   b) Can you draw a circle with area 20 cm^2?
   If your answer is yes, explain how you would do it.
   If your answer is no, explain why you cannot do it.

6. The table shows the type of land cover in Canada, as a percent of the total area.
   a) Draw a circle graph.
   b) Did you need to know the area of Canada to draw the circle graph? Explain.
   c) Write 3 things you know from looking at the graph.

Type of Land Cover in Canada

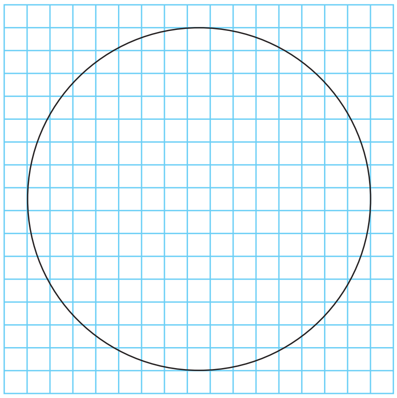
Forest and taiga	45%
Tundra	23%
Wetlands	12%
Fresh water	8%
Cropland and rangeland	8%
Ice and snow	3%
Human use	1%

Unit Problem Designing a Water Park

An anonymous donor gave a large sum of money to the Parks and Recreation Department.
The money is to be used to build a large circular water park.
Your task is to design the water park.

The water park has radius 30 m.

The side length of each square on this grid represents 4 m.



You must include the following features:

2 Wading Pools:

Each wading pool is triangular.
The pools do not have the same dimensions.
Each pool has area 24 m^2.

3 Geysers:

A geyser is circular.
Each geyser sprays water out of the ground,
and soaks a circular area with diameter 5 m or 10 m.

2 Wet Barricades:

A barricade has the shape of a parallelogram.
A row of nozzles in the barricade shoots water vertically.
The water falls within the area of the barricade.

4 Time-out Benches:

Each bench is shaped like a parallelogram.
It must be in the park.

At Least 1 Special Feature:

This feature will distinguish your park from other parks.
This feature can be a combination of any of the shapes you learned in this unit.
Give the dimensions of each special feature.
Explain why you included each feature in the park.

Check List

Your work should show:

- ✓ the area of each different shape you used
- ✓ a diagram of your design on grid paper
- ✓ an explanation of how you created the design
- ✓ how you calculated the area of the park that gets wet

Your teacher will give you a grid to draw your design.
You may use plastic shapes or cutouts to help you plan your park.
Complete the design.
Colour the design to show the different features.

Design your park so that a person can walk through the middle of the park without getting wet.
What area of the park will get wet?

Reflect on Your Learning

You have learned to measure different shapes.
When do you think you might use this knowledge outside the classroom?

Investigation

Digital Roots

Work with a partner.

The **digital root** of a number is the result of adding the digits of the number until a single-digit number is reached.

For example, the digital root of 27 is: $2 + 7 = 9$
To find the digital root of 168:
Add the digits: $1 + 6 + 8 = 15$
Since 15 is not a single-digit number, add the digits: $1 + 5 = 6$
Since 6 is a single-digit number, the digital root of 168 is 6.

A digital root can also be found for the product of a multiplication fact.
For the multiplication fact, 8×4:
$8 \times 4 = 32$
Add the digits in the product: $3 + 2 = 5$
Since 5 is a single-digit number, the digital root of 8×4 is 5.

You will explore the digital roots of the products in a multiplication table, then display the patterns you find.
As you complete the *Investigation*, include all your work in a report that you will hand in.

Materials:
- multiplication chart
- compass
- protractor
- ruler

Part 1

➤ Use a blank 12×12 multiplication chart.
Find each product.
Find the digital root of each product.
Record each digital root in the chart.
For example, for the product $4 \times 4 = 16$,
the digital root is $1 + 6 = 7$.

×	1	2	3	4	…
1					
2					
3					
4				7	
⋮					

➤ Describe the patterns in the completed chart.
Did you need to calculate the digital root of each product?
Did you use patterns to help you complete the table?
Justify the method you used to complete the chart.

➤ Look down each column. What does each column represent?

Part 2

➤ Use a compass to draw 12 circles.
Use a protractor to mark 9 equally spaced points on each circle.
Label these points in order, clockwise, from 1 to 9.
Use the first circle.
Look at the first two digital roots in the 1st column of your chart.
Find these numbers on the circle.
Use a ruler to join these numbers with a line segment.
Continue to draw line segments to join points that match the digital roots in the 1st column.
What shape have you drawn?

➤ Repeat this activity for each remaining column.
Label each circle with the number at the top of the column.

➤ Which circles have the same shape?
Which circle has a unique shape?
What is unique about the shape?
Why do some columns have the same pattern of digital roots?
Explain.

Take It Further

➤ Investigate if similar patterns occur in each case:
- Digital roots of larger 2-digit numbers, such as 85 to 99
- Digital roots of 3-digit numbers, such as 255 to 269

Write a report on what you find out.