## UNIT

1

## Patterns and Relations

Students in a Grade 7 class were raising money for charity.
Some students had a "bowl-a-thon."
This table shows the money that one student raised for different bowling times.

| Time (h) | Money Raised (\$) |
| :---: | :---: |
| 1 | 8 |
| 2 | 16 |
| 3 | 24 |
| 4 | 32 |
| 5 | 40 |
| 6 | 48 |



- What patterns do you see in the table?
- Extend the table.

For how long would the student have to bowl to raise $\$ 72$ ?

- Use patterns to explore divisibility rules.
- Translate between patterns and equivalent linear relations.
- Evaluate algebraic expressions by substitution.
- Represent linear relations in tables and graphs.
- Solve simple equations, then verify the solutions.


## Why It's

## Important

- Divisibility rules help us find the factors of a number.
- Graphs provide information and are a useful problem-solving tool.
- Efficient ways to represent a pattern can help us describe and solve problems.



## Key Words

- divisibility rules
- algebraic expression
- numerical coefficient
- constant term
- relation
- linear relation
- unit tile
- variable tile
- algebra tiles

Which of these numbers are divisible by 2? By 5? By 10?
How do you know?

- 78
- 27
- 35
- 410
- 123
- 2100
- 4126
- 795


## Explore

You will need a hundred chart numbered 301-400, and three different coloured markers.
> Use a marker. Circle all numbers on the hundred chart that are divisible by 2. Use a different marker.
Circle all numbers that are divisible by 4. Use a different marker.
Circle all numbers that are divisible by 8.
Describe the patterns you see in the numbers you circled.
> Choose 3 numbers greater than 400.
Which of your numbers do you think are divisible by 2? By 4? By 8?
Why do you think so?

## Reflect \& Share

Share your work with another pair of classmates.
Suppose a number is divisible by 8.
What else can you say about the number?
Suppose a number is divisible by 4.
What else can you say about the number?

## Connect

We know that 100 is divisible by 4 : $100 \div 4=25$
So, any multiple of 100 is divisible by 4 .
To find out if any whole number with 3 or more digits is divisible by 4 , we only need to check the last 2 digits.

To find out if 352 is divisible by 4 , check if 52 is divisible by 4 .
$52 \div 4=13$
52 is divisible by 4 , so 352 is divisible by 4 .
To check if a number, such as 1192 , is divisible by 8 ,
think: $1192=1000+192$
We know 1000 is divisible by $8: 1000 \div 8=125$
So, we only need to check if 192 is divisible by 8 .
Use mental math. $192 \div 8=24$
192 is divisible by 8 , so 1192 is divisible by 8 .

All multiples of 1000 are divisible by 8 .
So, for any whole number with 4 or more digits, we only need to check the last 3 digits to find out if the number is divisible by 8 .

A number that is divisible by 8 is also divisible by 2 and by 4 because $8=2 \times 4$.
So, a number divisible by 8 is even.

Another way to check if a number is divisible by 8 is to divide by 4 . If the quotient is even, then the number is divisible by 8 .

You can use patterns to find divisibility rules for other numbers.

- All multiples of 10 , such as 30,70 , and 260 , end in 0.

Any number whose ones digit is 0 , is divisible by 10 .
> Here are some multiples of 5 .
$5,10,15,20,25,30,35,40, \ldots, 150,155,160, \ldots$
The ones digits form a repeating pattern.
The core of the pattern is: 5,0
Any number whose ones digit is 0 or 5 , is divisible by 5 .

| 1 | (2) | 3 | (4) | 5) | (6) | 7 | (8) | 9 | (10) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 11 | (12) | 13 | (14) | (15) | 16 | 17 | (18) | 19 | (2) |
| 21 | (2) | 23 | (24) | (25) | (26) | 27 | (28) | 29 | 30 |
| 31 | (32) | 33 | (34) | (35) | (36) | 37 | (38) | 39 | (40) |
| 41 | (42) | 43 | (44) | (45) | (46) | 47 | (48) | 49 | (50) |
| 51 | (52) | 53 | (54) | (55) | (56) | 57 | (58) | 59 |  |
| 61 | (62) | 63 | 64) | (6) | 66 | 67 | 68) | 69 |  |
| 71 | (72) | 73 | (74) | (75) | 76 | 77 | (78) | 79 |  |
| 81 | (82) | 83 | (84) | 85) | 86 | 87 | (8) | 89 | (1) |
| 91 | (92) | 93 | (94) | (95) | (96) | 97 | (98) | 99 | 100 |

## Every multiple of 5 has a ones digit of 0 or 5 .

2 and 4 are factors of 8.

## Example

Which numbers are divisible by 5? By 8? Both by 5 and by 8 ?
How do you know?
$12,24,35,56,80,90,128,765,1048,1482,3960,15019$

## A Solution

Any number with 0 or 5 in the ones place is divisible by 5 .
So, the numbers divisible by 5 are: 35, 80, 90, 765, 3960
The divisibility rule for 8 only applies when a number is 1000 or greater.
For numbers less than 1000, use mental math or a calculator.
All multiples of 8 are even, so reject 35,765 , and 15019.
Use mental math to identify that 12 and 90 are not divisible by 8 .
Use mental math to identify that $24,56,80$, and 128 are divisible by 8 .
1048 and 3960 are divisible by 8 because 48 and 960 are divisible by 8 .
1482 is not divisible by 8 because 482 is not divisible by 8 .
We can display the results in a Venn diagram.


The numbers in the overlapping region are divisible both by 5 and by 8 .
So, 80 and 3960 are also divisible by 40 , since $5 \times 8=40$.

## Practice

1. Which numbers are divisible by 2? By 5?

How do you know?
a) 106
b) 465
c) 2198
d) 215
e) 1399
f) 4530
2. Explain why a number with 0 in the ones place is divisible by 10 .
3. Which numbers are divisible by 4? By 8? By 10? How do you know?
a) 212
b) 512
c) 5450
d) 380
e) 2132
f) 12256
4. Maxine and Tony discuss divisibility. Maxine says," 260 is divisible by 4 and by 5 .
$4 \times 5=20$, so 260 is also divisible by $20 .{ }^{\prime \prime}$
Tony says," 148 is divisible by 2 and by 4 .
$2 \times 4=8$, so 148 is also divisible by 8."
Are both Maxine and Tony correct? Explain your thinking.

5. Write 3 numbers that are divisible by 8 .

How did you choose the numbers?

## 6. Assessment Focus

a) Use the divisibility rules for 2,4 , and 8 to sort these numbers.

| 1046 | 322 | 460 | 1784 | 28 |
| :--- | :--- | :--- | :--- | :--- |
| 54 | 1088 | 224 | 382 | 3662 |

b) Draw a Venn diagram with 3 loops.

Label the loops: "Divisible by 2,""Divisible by 4," and "Divisible by 8"
Explain why you drew the loops the way you did.
Place the numbers in part a in the Venn diagram.
How did you decide where to place each number?
c) Find and insert 3 more 4-digit numbers in the Venn diagram.
7. Use the digits 0 to 9 . Replace the $\square$ in each number to make a number divisible by 4 . Find as many answers as you can.
a) $822 \square$
b) $2114 \square 8$
c) $15 \square 32$
8. Take It Further A leap year occurs every 4 years.

The years 1992 and 2004 were leap years.
What do you notice about these numbers?
Was 1964 a leap year? 1852? 1788? Explain.

## Reflect

Compare the divisibility rules for 4 and 8 .
How can you use one rule to help you remember the other?

Division can be thought of as making equal groups.

For $20 \div 4$, we make 4 equal groups of 5 .


## Explore

Use a calculator.
> Choose 10 different numbers.
Divide each number by 0 .
What do you notice?
What do you think this means?

- Choose 15 consecutive 2-digit numbers.

Divide each number by 3 and by 9 .
Repeat for 15 consecutive 3-digit numbers.
List the numbers that were divisible by 3 and by 9 .
Find the sum of the digits of each number.
What do you notice?
Choose 4 different 4-digit numbers you think are divisible by 3 and by 9 .
Divide each number by 3 and by 9 to check.
Add the digits in each number. What do you notice?
> Draw this Venn diagram.
Sort these numbers.
$\begin{array}{lllllllll}12 & 21 & 42 & 56 & 88 & 135 & 246 & 453 & 728\end{array}$
What can you say about the numbers in the overlapping region?


## Reflect \& Share

Share your work with another pair of classmates.
Explain how to choose a 4-digit number that is divisible by 3.
Without dividing, how can you tell if a number is divisible by 6 ? By 9 ?
Why do you think a number cannot be divided by 0 ?

## Connect

We can use divisibility rules to find the factors of a number, such as 100 .
Any number is divisible by 1 and itself, so 1 and 100 are factors of 100 . 100 is even, so 100 is divisible by 2.
We know 100 is divisible by 4 .
The ones digit is 0 , so 100 is divisible by 5 and by 10 .
100 is not divisible by $3,6,8$, or 9 .
$100 \div 1=100$
$100 \div 2=50$
$100 \div 4=25$
$100 \div 5=20$
$100 \div 10=10$

The factors of 100 , from least to greatest, are:
$1,2,4,5,10,20,25,50,100$

Factors occur in pairs.
When we find one factor of a number, we also find a second factor.
A whole number cannot be divided by 0 .
We cannot take a given number and share it into zero equal groups.
We cannot make sets of zero from a given number of items.

## Example

Edward has 16 souvenir miniature hockey sticks.
He wants to share them equally among his cousins.
How many sticks will each cousin get if Edward has:
a) 8 cousins?
b) 0 cousins?

Explain your answer to part b.

## A Solution

a) There are 16 sticks. Edward has 8 cousins.
$16 \div 8=2$
Each cousin will get 2 sticks.
b) There are 16 sticks. Edward has no cousins. 16 sticks cannot be shared equally among no cousins.
This answer means that we cannot divide a number by zero.
We cannot divide 16 by 0 because 16 cannot be shared into zero equal groups.

You have sorted numbers in a Venn diagram. You can also use a Carroll diagram to sort numbers.

Here is an example:

|  | Divisible by 3 | Not Divisible by 3 |
| :--- | :--- | :--- |
| Divisible by 8 | $24,120,1104,12096$ | $32,224,2360$ |
| Not Divisible by 8 | $12,252,819,11337$ | $10,139,9212$ |

## Divisibility Rules

A whole number is divisible by:
2 if the number is even
3 if the sum of the digits is divisible by 3
4 if the number represented by the last 2 digits is divisible by 4
5 if the ones digit is 0 or 5
6 if the number is divisible by 2 and by 3

## Practice

1. Which numbers are divisible by 3 ? By 9 ? How do you know?
a) 117
b) 216
c) 4125
d) 726
e) 8217
f) 12024
2. Write 3 numbers that are divisible by 6 . How did you choose the numbers?
3. Which of these numbers is 229344 divisible by? How do you know?
a) 2
b) 3
c) 4
d) 5
e) 6
f) 8
g) 9
h) 10
4. Use the divisibility rules to find the factors of each number. How do you know you have found all the factors?
a) 150
b) 95
c) 117
d) 80
5. Use a Carroll diagram.

Which numbers are divisible by 4? By 9 ? By 4 and by 9 ? By neither 4 or 9 ?

| 144 | 128 | 252 | 153 | 235 | 68 | 120 | 361 | 424 | 468 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

6. I am a 3-digit number that has a 2 in the hundreds place. I am divisible by 3,4 , and 5 . Which number am I?

## 7. Assessment Focus

a) Write a 3-digit number that is divisible by 5 and by 9 .

How did you choose the number?
b) Find the factors of the number in part a. Use the divisibility rules to help you.
c) How would you find the greatest 3-digit number that is divisible by 5 and by 9 ? The least 3-digit number? Explain your methods.
8. Use the digits 0 to 9 .

Replace the $\square$ in each number to make a number divisible by 3 .
Find as many answers as you can.
a) $4 \square 6$
b) $1 \square 32$
c) $2471 \square$
9. Suppose you have 24 cereal bars. You must share the bars equally with everyone in the classroom. How many cereal bars will each person get, in each case?
a) There are 12 people in the classroom.
b) There are 6 people in the classroom.
c) There is no one in the classroom.
d) Use your answer to part c.

Explain why a number cannot be divided by 0 .

10. Take It Further Universal Product Codes (UPCs) are used to identify retail products.

The codes have 12 digits, and sometimes start with 0.
To check that a UPC is valid, follow these steps:

- Add the digits in the odd-numbered positions (1st, 3rd, 5th,...).
- Multiply this sum by 3.
- To this product, add the digits in the even-numbered positions.
- The result should be a number divisible by 10.

Look at this UPC. Is it a valid code? Explain.
Find 2 UPC labels on products at home.
Check to see if the codes are valid. Record your results.


## Reflect

Which divisibility rules do you find easiest to use?
Which rules do you find most difficult? Justify your choices.

## Writing to Explain Your Thinking

Have you ever tried to explain how you solved a problem to a classmate?

Communicating your thinking can be difficult.
A Thinking Log can be used to record what and how you are thinking as you solve a problem. It is a good way to organize your thoughts.

A classmate, teacher, or parent should be able to follow your thinking to understand how you solved the problem.

## Using a Thinking Log

Complete a Thinking Log as you work through this problem.

Nine players enter the Saskatchewan
Thumb Wrestling Championship.
In the first round, each player wrestles every other player once.
How many matches are there in the first round?

Thinking Log Name:
I have been asked to find ...
Here's what I'll try first ...

To solve this problem I'll ...


And then...

Here's my solution...

## Reflect \& Share

- Read over what you have written. Will someone else be able to follow your thinking?
- Share your Thinking Log with a classmate. Was your classmate able to follow your thinking and understand your solution? Explain.
- Describe any changes you would make to improve your Thinking Log.



## Check

Complete a Thinking Log for each of these problems.

1. I am a 2-digit number. I have three factors.

I am divisible by five. Which number am I?
2. A book contains 124 pages numbered from 1 to 124 .

How many times does the digit 7 appear?
3. Here is a pattern of tiles.

a) How many tiles will there be in the 10th term?
b) Which term has 37 tiles? How do you know?

We can use symbols to represent a pattern.

## Explore

Tehya won some money in a competition.
She has two choices as to how she gets paid.
Choice 1: $\$ 20$ per week for one year
Choice 2: \$400 cash now plus \$12 per week for one year
Which method would pay Tehya more money?
For what reasons might Tehya choose each method of payment?

## Reflect \& Share



Work with another pair of classmates.
For each choice, describe a rule you can use to calculate the total money Tehya has received at any time during the year.

## Connect

We can use a variable to represent a number in an expression.
For example, we know there are 100 cm in 1 m .


We can write $1 \times 100 \mathrm{~cm}$ in 1 m .
There are $2 \times 100 \mathrm{~cm}$ in 2 m .
There are $3 \times 100 \mathrm{~cm}$ in 3 m .

Recall that a variable is a letter, such as $n$, that represents a quantity that can vary.

To write an expression for the number of centimetres in any number of metres, we say there are $n \times 100 \mathrm{~cm}$ in $n$ metres.
$n$ is a variable.
$n$ represents any number we choose.
We can use any letter, such as $n$ or $x$, as a variable.
The expression $n \times 100$ is written as $100 n$.

Variables are written in italics so they are not confused with units of measurement. $100 n$ is an algebraic expression.

Here are some other algebraic expressions, and their meanings.
In each case, $n$ represents the number.

- Three more than a number: $3+n$ or $n+3$
- Seven times a number: 7n
$7 n$ means $7 \times n$.
- Eight less than a number: $n-8$
- A number divided by $20: \frac{n}{20}$

When we replace a variable with a number in an algebraic expression, we evaluate the expression. That is, we find the value of the expression for a particular value of the variable.

## Example

Write each algebraic expression in words.
Then evaluate for the value of the variable given.
a) $5 k+2$ for $k=3$
b) $32-\frac{x}{4}$ for $x=20$

## A Solution

a) $5 k+2$ means 5 times a number, then add 2 .

Replace $k$ with 3 in the expression $5 k+2$.
Then use the order of operations.

$$
\begin{aligned}
5 k+2 & =5 \times 3+2 & & \text { Multiply first. } \\
& =15+2 & & \text { Add. } \\
& =17 & &
\end{aligned}
$$

b) $32-\frac{x}{4}$ means 32 minus a number divided by 4 .

Replace $x$ with 20 in the expression $32-\frac{x}{4}$.

```
\frac{X}{4}}\mathrm{ means }x\div4
```

Then use the order of operations.

$$
\begin{aligned}
32-\frac{x}{4} & =32-\frac{20}{4} & & \text { Divide first. } \\
& =32-5 & & \text { Subtract. } \\
& =27 & &
\end{aligned}
$$

In the expression $5 k+2$,

- 5 is the numerical coefficient of the variable.
- 2 is the constant term.
- $k$ is the variable.

The variable represents any number in a set of numbers.

## Practice

1. Identify the numerical coefficient, the variable, and the constant term in each algebraic expression.
a) $3 x+2$
b) $5 n$
c) $w+3$
d) $2 p+4$
2. An algebraic expression has variable $p$, numerical coefficient 7 , and constant term 9.
Write as many different algebraic expressions as you can that fit this description.
3. Write an algebraic expression for each phrase.
a) six more than a number
b) a number multiplied by eight
c) a number decreased by six
d) a number divided by four
4. A person earns $\$ 4$ for each hour he spends baby-sitting.
a) Find the money earned for each time.
i) 5 h
ii) 8 h
b) Write an algebraic expression you could use to find the money earned in $t$ hours.
5. Write an algebraic expression for each sentence.

a) Double a number and add three.
b) Subtract five from a number, then multiply by two.
c) Divide a number by seven, then add six.
d) A number is subtracted from twenty-eight.
e) Twenty-eight is subtracted from a number.
6. a) Write an algebraic expression for each phrase.
i) four more than a number
ii) a number added to four
iii) four less than a number
iv) a number subtracted from four
b) How are the expressions in part a alike?

How are they different?
7. Evaluate each expression by replacing $x$ with 4 .
a) $x+5$
b) $3 x$
c) $2 x-1$
d) $\frac{x}{2}$
e) $3 x+1$
f) $20-2 x$
8. Evaluate each expression by replacing $z$ with 7 .
a) $z+12$
b) $10-z$
c) $5 z$
d) $3 z-3$
e) $35-2 z$
f) $3+\frac{z}{7}$
9. Assessment Focus Jason works at a local fish and chips restaurant.
He earns $\$ 7 / \mathrm{h}$ during the week, and $\$ 9 / \mathrm{h}$ on the weekend.
a) Jason works 8 h during the week and 12 h on the weekend.
Write an expression for his earnings.
b) Jason works $x$ hours during the week and 5 h on the weekend.
Write an expression for his earnings.
c) Jason needs $\$ 115$ to buy sports equipment.

He worked 5 h on the weekend.
How many hours does Jason have to work during the week to have the money he needs?
10. Take It Further $A$ value of $n$ is substituted in each expression to get the number in the box.
Find each value of $n$.
a) $5 n$
30
b) $3 n-1$ 11
c) $4 n+7$
15
d) $5 n-4 \quad 11$
e) $4+6 n$
40
f) $\frac{n}{8}$ 5

## Reflect

Explain why it is important to use the order of operations when evaluating an algebraic expression. Use an example in your explanation.

Here is a pattern made from linking cubes.

A pattern rule is: Start at 3 . Add 3 each time.
This rule relates each term to the term that comes before it.

We can also describe this pattern using the term number.

| Term Number | 1 | 2 | 3 | 4 |
| :--- | :---: | :---: | :---: | :---: |
| Term | 3 | 6 | 9 | 12 |

How does each term relate to the term number?

## Explore



On Enviro-Challenge Day, Grade 7 classes compete to see which class can collect the most garbage.

Each student in Ms. Thomson's class pledges to pick up 6 pieces of garbage.

- How many pieces of garbage will be picked up when the number of students is 5? 10? 15? 20? 25? 30?
> What pattern do you see in the numbers of pieces of garbage?
- Write a rule to find how many pieces of garbage will be picked up, when you know the number of students.
> Write an algebraic expression for the number of pieces of garbage picked up by $n$ students.



## Reflect \& Share

Share your work with another pair of classmates.
Find the number of pieces of garbage picked up by 35 students.
How can you do this using the pattern?
Using the rule? Using the algebraic expression?

## Connect

Miss Jackson's class pledges to pick up a total of 10 more pieces of garbage than Ms. Thomson's class. Here are the numbers of pieces of garbage picked up by different numbers of students.

| Number of students | 2 | 4 | 6 | 8 | 10 | 12 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of pieces of <br> garbage picked up by <br> Ms. Thomson's class | 12 | 24 | 36 | 48 | 60 | 72 |
| Number of pieces of <br> garbage picked up by <br> Miss Jackson's class | 22 | 34 | 46 | 58 | 70 | 82 |


| Pieces of garbage |  |
| :--- | :--- | :--- |
| picked up by | Pieces of garbage |
| Miss Jackson's class |  |

Let $n$ represent the number of students who pick up garbage in Ms. Thomson's class.
Then the number of pieces of garbage picked up by Ms. Thomson's class is $6 n$.
And, the number of pieces of garbage picked up
by Miss Jackson's class is $10+6 n$.

Recall that 10 is the constant term.

The number of pieces of garbage is related to the number of students.
When we compare or relate a variable to an expression that contains the variable, we have a relation.
That is, $10+6 n$ is related to $n$. This is a relation.

## Example

Mr. Prasad plans to hold a party for a group of his friends. The cost of renting a room is $\$ 35$.
The cost of food is $\$ 4$ per person.
a) Write a relation for the cost of the party, in dollars, for $n$ people.
b) How much will a party cost for 10 people?
 For 15 people?
c) How does the relation change if the cost of food doubles?

How much more would a party for 10 people cost?
How do you know the answer makes sense?

## A Solution

a) The cost of renting a room is $\$ 35$.

This does not depend on how many people come.
The cost of food is $\$ 4$ per person.
If 5 people come, the cost of food in dollars is: $4 \times 5=20$
If $n$ people come, the cost of food in dollars is: $4 \times n$, or $4 n$
So, $n$ is related to $35+4 n$.
b) To find the cost for 10 people, substitute $n=10$ into $35+4 n$.

$$
\begin{aligned}
35+4 n & =35+4(10) \\
& =35+40 \\
& =75
\end{aligned}
$$

$4(10)$ means $4 \times 10$.

The party will cost $\$ 75$.
To find the cost for 15 people, substitute $n=15$ into $35+4 n$.

$$
\begin{aligned}
35+4 n & =35+4(15) \\
& =35+60 \\
& =95
\end{aligned}
$$

The party will cost $\$ 95$.
c) If the cost of food doubles, Mr. Prasad will pay $\$ 8$ per person.

If $n$ people come, the cost for food, in dollars, is 8 .
For $n$ people, the cost of the party, in dollars, is now $35+8 n$.
If 10 people come, the cost is now:

$$
\begin{aligned}
35+8 n & =35+8(10) \\
& =35+80 \\
& =115
\end{aligned}
$$

The party will cost $\$ 115$.
This is an increase of $\$ 115-\$ 75=\$ 40$.
The answer makes sense because the cost is now $\$ 4$ more per person.
So, the extra cost for 10 people would be $\$ 4 \times 10$, or $\$ 40$ more.


## Linal $_{1}$

History
The word "algebra" comes from the Arabic word "al-jabr." This word appeared in the title of one of the earliest algebra texts, written around the year 825 by al-Khwarizmi. He lived in what is now Uzbekistan.

## Practice

1. i) For each number pattern, how is each term related to the term number?
ii) Let $n$ represent any term number. Write a relation for the term.

| a) | Term Number | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Term | 2 | 4 | 6 | 8 | 10 | 12 |
| b) | Term Number | 1 | 2 | 3 | 4 | 5 | 6 |
|  | Term | 3 | 4 | 5 | 6 | 7 | 8 |
| c) | Term Number | 1 | 2 | 3 | 4 | 5 | 6 |
|  | Term | 8 | 16 | 24 | 32 | 40 | 48 |
| d) | Term Number | 1 | 2 | 3 | 4 | 5 | 6 |
|  | Term | 6 | 7 | 8 | 9 | 10 | 11 |

2. There are $n$ students in a class. Write a relation for each statement.
a) the total number of pencils, if each student has three pencils
b) the total number of desks, if there are two more desks than students
c) the total number of geoboards, if each pair of students shares one geoboard
d) the total number of stickers, if each student gets four stickers and there are ten stickers left over
3. A person earns $\$ 10$ for each hour worked.
a) Write a relation for her earnings for $n$ hours of work.
b) How much does she earn for 30 h of work?
4. a) Write a relation for the perimeter of a square with side length $n$ centimetres.
b) What is the perimeter of a square with side length 12 cm ?

c) Suggest a situation that could be represented by each relation.
i) $3 s$ is related to $s$
ii) $8 t$ is related to $t$
5. Suggest a real-life situation that could be represented by each relation.
a) $n+5$ is related to $n$
b) $15+2 p$ is related to $p$
c) $3 t+1$ is related to $t$

How do you know each situation fits the relation?
6. Koko is organizing an overnight camping trip. The cost to rent a campsite is $\$ 20$. The cost of food is $\$ 9$ per person.
a) How much will the trip cost if 5 people go? 10 people go?
b) Write a relation for the cost of the trip when $p$ people go.
c) Suppose the cost of food doubles.

Write a relation for the total cost of the trip for $p$ people.
d) Suppose the cost of the campsite doubles. Write a relation for the total cost of the trip for $p$ people.
e) Explain why using the variable $p$ is helpful.

7. Assessment Focus A pizza with cheese and tomato toppings costs $\$ 8.00$. It costs \$1 for each extra topping.
a) Write a relation for the cost of a pizza with e extra toppings.
b) What is the cost of a pizza with 5 extra toppings?
c) On Tuesdays, the cost of the same pizza with cheese and tomato toppings is $\$ 5.00$. Write a relation for the cost of a pizza with e extra toppings on Tuesdays.
d) What is the cost of a pizza with 5 extra toppings on Tuesdays?
e) How much is saved by buying the pizza on Tuesday?

8. Write a relation for the pattern rule for each number pattern.

Let $n$ represent any term number.
a) $4,8,12,16, \ldots$
b) $7,8,9,10, \ldots$
c) $0,1,2,3, \ldots$
9. Take It Further
i) For each number pattern, how is each term related to the term number?
ii) Let $n$ represent any term number. Write a relation for the term.

| a) | Term Number | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Term | 3 | 5 | 7 | 9 | 11 | 13 |
| b) | Term Number | 3 | 4 | 5 | 6 | 7 | 8 |
|  | Term | 7 | 10 | 13 | 16 | 19 | 22 |
| c) | Term Number | 2 | 3 | 4 | 5 | 6 | 7 |
|  | Term | 5 | 9 | 13 | 17 | 21 | 25 |

## Reflect

How did your knowledge of patterning help you in this lesson?

## 1.5 Patterns and Relationships in Tables

An Input/Output machine represents a relation. Any Input number can be represented by $n$.

Suppose you input $n=8$.
What will the output be?
How is the output related to the input?


## Explore

Sketch an Input/Output machine like this one.


Write an algebraic expression to go in the machine.
> Use the numbers 1 to 6 as input.
Find the output for each Input number.
Record the input and output in a table like this.
> How is the output related to the input?
> Describe the pattern in the Output numbers.

| Input | Output |
| :---: | :---: |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |

## Reflect \& Share

Share your work with another pair of classmates.
Describe how you would find the next 3 Output numbers for your classmates' Input/Output machine.
How is the output related to the input?

## Connect

This Input/Output machine relates $n$ and $2 n+3$.
To create a table of values, select a set of Input numbers.
To get each Output number, multiply the

$$
\begin{aligned}
& \text { When } n=1,2 n+3 \\
& =2(1)+3 \\
& \\
& =2+3 \\
& \\
& =5 \\
& \text { When } n=2,2 n+3
\end{aligned}=2(2)+3 \text {. } \begin{aligned}
& =4+3 \\
& =7 \\
\text { When } n=3,2 n+3 & =2(3)+3 \\
& =6+3 \\
& =9
\end{aligned}
$$

Remember the order of operations. Multiply before adding.

and so on.

We used consecutive Input numbers.
The Output numbers form a pattern. They increase by 2 each time.
This is because the expression contains $2 n$, which means that the Input number is doubled.
When the Input number increases by 1 , the Output number increases by 2.

The expression $2 n+3$ can also be written as $3+2 n$.
When a relation is represented as a table of values, we can write the relation using algebra.

## Example

Write the relation represented by this table.

| Input | Output |
| :---: | :---: |
| 1 | 2 |
| 2 | 5 |
| 3 | 8 |
| 4 | 11 |
| 5 | 14 |

## A Solution

Let any Input number be represented by $n$.
The input increases by 1 each time.
The output increases by 3 each time.
This means that the expression for the output contains $3 n$.

Substitute several values of $n$ in $3 n$, then look for a pattern.

When $n=1,3 n=3(1)=3$
When $n=2,3 n=3(2)=6$
When $n=3,3 n=3(3)=9$
When $n=4,3 n=3(4)=12$
When $n=5,3 n=3(5)=15$


Each value is 1 more than the output above.
That is, the output is
1 less than each value.

So, the output is $3 n-1$.
The table shows how $3 n-1$ relates to $n$.

## Another Solution

Another way to solve this problem is to notice that each output is 1 less than a multiple of 3 .
So, the output is $3 \times n-1$, or $3 n-1$.
The table shows how $3 n-1$ relates to $n$.

| Input | Output |
| :---: | :---: |
| 1 | $2=3 \times 1-1$ |
| 2 | $5=3 \times 2-1$ |
| 3 | $8=3 \times 3-1$ |
| 4 | $11=3 \times 4-1$ |
| 5 | $14=3 \times 5-1$ |
| $n$ | $3 \times n-1$ |

## Practice

1. Copy and complete each table.

Explain how the Output number is related to the Input number.
a)

| Input | Output |
| :---: | :---: |
| $x$ | $2 x$ |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |

b)

| Input <br> $m$ | Output <br> $10-m$ |
| :---: | :---: |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |

c)

| Input | Output |
| :---: | :---: |
| $p$ | $3 p+5$ |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |

2. Use algebra. Write a relation for each Input/Output table.
a)

| Input <br> $n$ | Output |
| :---: | :---: |
| 1 | 7 |
| 2 | 14 |
| 3 | 21 |
| 4 | 28 |

b)

| Input <br> $n$ | Output |
| :---: | :---: |
| 1 | 4 |
| 2 | 7 |
| 3 | 10 |
| 4 | 13 |

c)

| Input <br> $n$ | Output |
| :---: | :---: |
| 1 | 1 |
| 2 | 3 |
| 3 | 5 |
| 4 | 7 |

3. Assessment Focus For each table, find the output.

Explain how the numbers 3 and 4 in each relation affect the output.
a)

| Input | Output |
| :---: | :---: |
| $n$ | $3 n+4$ |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |

b)

| Input | Output |
| :---: | :---: |
| $n$ | $4 n+3$ |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |

4. Use algebra. Write a relation for each Input/Output table.
a)

| Input <br> $x$ | Output |
| :---: | :---: |
| 1 | 5 |
| 2 | 8 |
| 3 | 11 |
| 4 | 14 |

b)

| Input <br> $x$ | Output |
| :---: | :---: |
| 1 | 1 |
| 2 | 7 |
| 3 | 13 |
| 4 | 19 |

c)

| Input <br> $x$ | Output |
| :---: | :---: |
| 1 | 8 |
| 2 | 13 |
| 3 | 18 |
| 4 | 23 |

5. Take It Further
a) Describe the patterns in this table.
b) Use the patterns to extend the table 3 more rows.
c) Use algebra.

Write a relation that describes how the output is related to the input.

| Input <br> $x$ | Output |
| :---: | :---: |
| 5 | 1 |
| 15 | 3 |
| 25 | 5 |
| 35 | 7 |
| 45 | 9 |
| 55 | 11 |

## Reflect

Your friend missed today's lesson. Explain how to write the relation represented by an Input/Output table.

## Mid-Unit Review

## LESSON

1.1 1. Which numbers are divisible by 4? By 8? How do you know?
a) 932
b) 1418
c) 5056
d) 12160
e) 14436
2. Draw a Venn diagram with 2 loops. Label the loops: "Divisible by 3" and "Divisible by 5." Sort these numbers: 5485123735174037566195 What is true about the numbers in the overlapping region?
3. Use the divisibility rules.

Find the factors of each number.
a) 85
b) 136
c) 270
4. Write an algebraic expression for each statement.
Let $n$ represent the number.
a) seven more than a number
b) a number multiplied by eleven
c) a number divided by six
d) three less than four times a number
e) the sum of two and five times a number
5. Predict which expression in each pair will have the greater value when $y$ is replaced with 8 .
Evaluate to check your predictions.
a) i) $y+7$
ii) $2 y$
b) i) $6 y$
ii) $9-y$
c) i) $\frac{y+4}{2}$
ii) $\frac{y}{2}+4$
d) i) $2 y+6$
ii) $3 y-6$
6. i) For each number pattern, how is each term related to the term number?
ii) Let $n$ represent the term number. Write a relation for the term.
a)

| Term Number | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Term | 6 | 12 | 18 | 24 | 30 | 36 |

b)

| Term Number | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Term | 5 | 6 | 7 | 8 | 9 | 10 |

7. Dave pays to practise in a music studio. He pays $\$ 12$ each month, plus $\$ 2$ for each hour he practises.
a) Write a relation for the total cost for one month, in dollars, when Dave practises $t$ hours.
b) How much will Dave pay to practise 10 h in one month? 20 h ?
c) How does the relation change when the cost per hour doubles?
8. Use algebra. Write a relation for each Input/Output table.
a)

| Input <br> $x$ | Output |
| :---: | :---: |
| 1 | 7 |
| 2 | 11 |
| 3 | 15 |
| 4 | 19 |

b) Input Output

| $x$ |  |
| :---: | :---: |
| 1 | 5 |
| 2 | 13 |
| 3 | 21 |
| 4 | 29 |

## 1.6 <br> Graphing Relations

Focus Create a table of values, then graph.

We can use a graph to show the relationship between two quantities.
What does this graph show?

How many jellybeans are in each bag?
Write a relation for the total number of jellybeans in $n$ bags.


## Explore

You will need grid paper.
The cost of $n$ CDs, in dollars, is $12 n$.

- What is the cost of one CD?
> Copy and complete this table.
> Graph the data.
Use the graph to answer these questions:
- What is the cost of 5 CDs?
- How many CDs could you buy with $\$ 72$ ?

| Number of <br> CDs <br> $n$ | Cost <br> (\$) <br> $12 n$ |
| :---: | :---: |
| 0 |  |
| 2 |  |
| 4 |  |
| 6 |  |
| 8 |  |
| 10 |  |



## Reflect \& Share

Describe the patterns in the table. How are these patterns shown in the graph?
If you had $\$ 50$, how many CDs could you buy?

## Connect

This table shows how $4 n+2$ relates to $n$, where $n$ is a whole number. We could have chosen any Input numbers, but to see patterns it helps to use consecutive numbers.

These data are plotted on a graph.
The input is plotted on the horizontal axis.
The output is plotted on the vertical axis.

| $\begin{gathered} \text { Input } \\ n \end{gathered}$ | Output $4 n+2$ |
| :---: | :---: |
| 0 | $4(0)+2=2$ |
| 1 | $4(1)+2=6$ |
| 2 | $4(2)+2=10$ |
| 3 | $4(3)+2=14$ |
| 4 | $4(4)+2=18$ |
| 5 | $4(5)+2=22$ |

On the vertical axis, the scale is 1 square for every 2 units.
The graph also shows how $4 n+2$ relates to $n$.

When we place a ruler along the points, we see the graph is a set of points that lie on a straight line. When points lie on a straight line, we say the relation is a linear relation.

Since no numbers lie between the Input values in the table, it is not meaningful to join the points with a solid line.

The graph shows that each time the input increases by 1 , the output increases by 4 .

Graph of $4 n+2$ against $n$


## Example

Mr. Beach has 25 granola bars.
He gives 3 granola bars to each student who stays after school to help prepare for the school concert.
a) Write a relation to show how the number of granola bars that remain is related to the number of helpers.
b) Make a table to show this relation.
c) Graph the data. Describe the graph.
d) Use the graph to answer these questions:
i) How many granola bars remain when 7 students help?
ii) When will Mr. Beach not have enough granola bars?


## A Solution

a) Let $n$ represent the number of helpers. Each helper is given 3 granola bars. So, the number of granola bars given to $n$ helpers is $3 n$.
There are 25 granola bars.
The number of granola bars that remain is $25-3 n$.
So, $n$ is related to $25-3 n$.
c) On the vertical axis, use a scale of 1 square for every 2 units.
The points lie on a line so the graph represents a linear relation.
When the input increases by 1 , the output decreases by 3 .
The graph goes down to the right.
This is because the number of granola bars that remain decreases as the number of helpers increases.
d) i) To find the number of granola bars that remain, extend the graph. The points lie on a straight line. Extend the graph to 7 helpers. There are 4 granola bars left.
ii) Continue to extend the graph. 25 granola bars are enough for 8 helpers, but not for 9 helpers. Mr. Beach will not have enough granola bars for 9 or more helpers.
b) Substitute each value of $n$ into $25-3 n$.

| Number of <br> Helpers <br> $\boldsymbol{n}$ | Number of <br> Granola Bars Left <br> $\mathbf{2 5 - 3 \boldsymbol { n }}$ |
| :---: | :---: |
| 0 | $25-3(0)=25$ |
| 1 | $25-3(1)=22$ |
| 2 | $25-3(2)=19$ |
| 3 | $25-3(3)=16$ |
| 4 | $25-3(4)=13$ |
| 5 | $25-3(5)=10$ |




To graph a relation, follow these steps:

- Select appropriate Input numbers. Make a table of values.
- Choose scales for the horizontal and vertical axes.
- Use a ruler to draw the axes on grid paper. Use numbers to indicate the scale.
- Label the axes. Give the graph a title.
- Plot the data in the table.


## Practice

1. Copy and complete this Input/Output table for each relation.
a) $4 n$ is related to $n$.
b) $x+3$ is related to $x$.
c) $4 c+6$ is related to $c$.
2. Graph each relation in question 1.

Suggest a real-life situation it could represent.
3. a) Copy and complete this Input/Output table to show how $6 a-4$ is related to $a$.
b) Graph the relation.

What scale did you use on the vertical axis?
How did you make your choice.
c) Explain how the graph illustrates the relation.
4. Look at the graph on the right.
a) What is the output when the input is 1 ?
b) Which input gives the output 18?
c) Extend the graph. What is the output when the input is 8 ?
d) Suggest a real-life situation this graph could represent.
5. Admission to Fun Place is $\$ 5$.

Each go-cart ride costs an additional \$3.
a) Write a relation to show how the total cost is related to the number of go-cart rides.
b) Copy and complete this table.
c) Draw a graph to show the relation.

Describe the graph.
d) Use the graph to answer these questions:
i) Erik goes on 6 go-cart rides.

What is his total cost?

| Number of <br> Go-Cart <br> Rides | Total <br> Cost <br> (\$) |
| :---: | :---: |
| 0 |  |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |

ii) Before entering the park, Lydia has $\$ 30$. How many go-cart rides can she afford?
6. Match each graph to its relation.
a) The number of seashells collected is related to the number of students who collected.
There are 12 seashells to start.
Each student collects 3 seashells.
b) The number of counters on the teacher's desk is related to the number of students who remove counters.
There are 36 counters to start.
Each student removes 6 counters.
c) The money earned baby-sitting is related to the number
 of hours worked. The baby-sitter earns $\$ 6 / \mathrm{h}$.
i)

ii)

iii)

7. Akuti borrows $\$ 75$ from her mother to buy a new lacrosse stick. She promises to pay her mother \$5 each week until her debt is paid off.
a) Write a relation to show how the amount Akuti owes is related to the number of weeks.
b) Make a table for the amount owing after $2,4,6,8$, and 10 weeks.
c) Draw a graph to show the relation. Describe the graph.
d) Use the graph to answer these questions:
i) How much does Akuti owe her mother after 13 weeks?
ii) When will Akuti finish paying off her debt?
8. Assessment Focus Use the relation: $5 n+6$ is related to $n$
a) Describe a real-life situation that could be represented by this relation.
b) Make a table of values using appropriate Input numbers.
c) Graph the relation. Describe the graph.
d) Write 2 questions you could answer using the graph. Answer the questions.

## Reflect

How can the graph of a relation help you answer questions about the relation? Use an example to show your thinking.

## 1.7 Reading and Writing Equations

## Explore

## Part 1

- Write an algebraic expression for these statements:

Think of a number.
Multiply it by 3 .
Add 4.
> The answer is 13 . What is the original number?

## Part 2

- Each of you writes your own number riddle.

Trade riddles with your partner.
> Write an algebraic expression for your partner's statements.
Find your partner's original number.

## Reflect 8. Share

Compare your answer to Part 1 with that of another pair of classmates.
If you found different values for the original number, who is correct?
Can both of you be correct? How can you check?

## Connect

Zena bought 3 CDs.
Each CD costs the same amount.
The total cost was \$36.
What was the cost of 1 CD?


We can write an equation for this situation. Let $p$ dollars represent the cost of 1 CD. Then, the cost of 3 CDs is $3 p$. This is equal to $\$ 36$.
We can write an equation to represent this situation: $3 p=36$


When we write one quantity equal to another quantity, we have an equation. Each quantity may be a number or an algebraic expression.
For example, $3 x+2$ is an algebraic expression; 11 is a number.
When we write $3 x+2=11$, we have an equation.
An equation is a statement that two quantities are equal.
Each side of the equation has the same value.

In an equation, the variable represents a specific unknown number.
When we find the value of the unknown number, we solve the equation.

## Example

Write an equation for each sentence.
a) Three more than a number is 15 .
b) Five less than a number is 7 .
c) A number subtracted from 5 is 1 .
d) A number divided by 3 is 10 .
e) Eight added to 3 times a number is 26 .

## A Solution

a) Three more than a number is 15 . Let $x$ represent the number.
Three more than $x: x+3$
The equation is: $x+3=15$
c) A number subtracted from 5 is 1 .

Let $g$ represent the number.
$g$ subtracted from 5: $5-g$
The equation is: $5-g=1$
e) Eight added to 3 times a number is 26 .

Let $h$ represent the number.
3 times $h$ : $3 h$
8 added to $3 h: 3 h+8$
The equation is: $3 h+8=26$

## Practice

1. Write an equation for each sentence.
a) Eight more than a number is 12 .
b) Eight less than a number is 12 .
2. Write a sentence for each equation.
a) $12+n=19$
b) $3 n=18$
c) $12-n=5$
d) $\frac{n}{2}=6$
3. Write an equation for each sentence.
a) Six times the number of people in the room is 258 .
b) One-half the number of students in the band is 21 .
c) The area of a rectangle with base 6 cm and height $h$ centimetres is $36 \mathrm{~cm}^{2}$.
4. The perimeter of a square is 156 cm . Write an equation you could use to find the side length of the square.


Recall that perimeter is the distance around a shape.
5. The side length of a regular hexagon is 9 cm . Write an equation you could use to find the perimeter of the hexagon.

6. Match each equation with the correct sentence.
a) $n+4=8$
A. Four less than a number is 8.
b) $4 n=8$
B. Four more than four times a number is 8 .
c) $n-4=8$
C. The sum of four and a number is 8 .
d) $4+4 n=8$
D. The product of four and a number is 8 .
7. Alonso thinks of a number.

He divides the number by 4 , then adds 10 .
The answer is 14.
Write an equation for the problem.

## 8. Assessment Focus

a) Write an equation for each sentence.
i) Five times the number of students is 295.
ii) The area of a rectangle with base 7 cm and height $h$ centimetres is $28 \mathrm{~cm}^{2}$.
iii) The cost of 2 tickets at $x$ dollars each and 5 tickets at $\$ 4$ each is $\$ 44$.
iv) Bhavin's age 7 years from now will be 20 years old.
b) Which equation was the most difficult to write? Why?
c) Write your own sentence, then write it as an equation.


## Reflect

Give an example of an algebraic expression and of an equation.
How are they similar? How are they different?

We can use tiles to represent an expression.
One yellow tile $\square$ can represent +1 .
We call it a unit tile.

We also use tiles to represent variables.
This tile represents $x$. $\square$
We call it an $x$-tile, or a variable tile.

A unit tile and a variable tile are collectively algebra tiles.

What algebraic expression do these tiles represent?


In this lesson, you will learn how to use tiles to solve equations. In Unit 6, you will learn other ways to solve equations.

## Explore

Alison had \$13.
She bought 5 gift bags.
Each bag costs the same amount.
Alison then had \$3 left.
How much was each gift bag?
> Let d dollars represent the cost of 1 gift bag. Write an equation to represent the problem.
> Use tiles. Solve the equation to find the value of $d$.
 How much was each gift bag?

## Reflect \& Share

Compare your equation with that of another pair of classmates.
If the equations are different, try to find out why.
Discuss your strategies for using tiles to solve the equation.

## Connect

Owen collects model cars.
His friend gives him 2 cars.
Owen then has 7 cars.
How many cars did he have at the start?
We can write an equation that we can solve to find out. Let $x$ represent the number of cars Owen had at the start.
2 more than $x$ is: $x+2$
The equation is: $x+2=7$

We can use tiles to solve this equation.
We draw a vertical line in the centre of the page.
It represents the equals sign in the equation.

We arrange tiles on each side of the line to represent the expression or number on each side of the equation.

We want to get the $x$-tile on its own.
This is called isolating the variable.
When we solve an equation, we must preserve the equality.
That is, whatever we do to one side of the equation, we must also do to the other side.

To solve the equation $x+2=7$ :
On the left side, put tiles On the right side, put tiles to represent $x+2$.


To isolate the $x$-tile, remove the 2 unit tiles from the left side.
To preserve the equality, remove 2 unit tiles from the right side, too.


The tiles show the solution is $x=5$.


To verify the solution, replace $x$ with 5 yellow tiles.


Right side: $\square \square \square \square 7$ yellow tiles

Since the left side and right side have equal numbers of tiles, the solution $x=5$ is correct.
Owen had 5 cars at the start.

## Example

Two more than three times a number is 14 .
a) Write an equation you can solve to find the number.
b) Use tiles to solve the equation.
c) Verify the solution.

## A Solution

a) Two more than three times a number is 14 .

Let $x$ represent the number.
Three times $x$ : $3 x$
Two more than $3 x: 3 x+2$
The equation is: $3 x+2=14$
b) $3 x+2=14$


Remove 2 unit tiles from each side to isolate the $x$-tiles.


There are $3 x$-tiles.
Arrange the tiles remaining on each side into 3 equal groups.


One $x$-tile equals 4 unit tiles.
So, $x=4$
c) To verify the solution, replace $x$ with 4 yellow tiles.


Since the left side and right side have equal numbers of tiles, the solution $x=4$ is correct.

## Practice

Use tiles to solve each equation.

1. Draw pictures to represent the steps you took to solve each equation.
a) $x+6=13$
b) $4+x=12$
c) $11=x+7$
d) $2 x=16$
e) $18=3 x$
f) $4 x=12$
2. Seven more than a number is 12 .
a) Write an equation for this sentence.
b) Solve the equation. Verify the solution.
3. For each equation in question 1 , identify a constant term, the numerical coefficient, and the variable.
4. At the used bookstore, one paperback book costs $\$ 3$.

How many books can be bought for $\$ 12$ ?
a) Write an equation you can solve to find how many books can be bought.
b) Solve the equation. Verify the solution.
5. Kiera shared 20 hockey cards equally among her friends.

Each friend had 4 cards.
a) Write an equation that describes this situation.
b) Solve the equation to find how many friends shared the cards.
6. In Nirmala's Grade 7 class, 13 students walk to school. There are 20 students in the class.
a) Write an equation you can solve to find how many students do not walk to school.
b) Solve the equation. Verify the solution.
7. Jacob is thinking of a number. He multiplies it by 3 and then adds 4 . The result is 16 .
a) Write an equation to represent this situation.
b) Solve the equation to find Jacob's number.
8. Assessment Focus Tarana had 2 paper plates. She bought 4 packages of paper plates. Each package had the same number of plates. Tarana now has a total of 18 plates. How many paper plates were in each package?
a) Write an equation you can solve to find how many plates were in each package.
b) Solve the equation. Verify the solution.
9. Take It Further Dominique has 20 comic books. She gives 5 to her sister, then gives 3 to each of her friends. Dominique has no comic books left.
a) Write an equation you can solve to find how many friends were given comic books.
b) Solve the equation. Verify the solution.
10. Take It Further
a) Write an equation whose solution is $x=4$.
b) Write a sentence for your equation.
c) Solve the equation.
d) Describe a situation that can be represented by your equation.

## Reflect

When you solve an equation, how can you be sure that your solution is correct?

## Unit Review

## What Do I Need to Know?

$\checkmark$
A whole number is divisible by:
2 if the number is even
3 if the sum of the digits is divisible by 3
4 if the number represented by the last 2 digits is divisible by 4
5 if the ones digit is 0 or 5
6 if the number is divisible by 2 and by 3
8 if the number represented by the last 3 digits is divisible by 8
9 if the sum of the digits is divisible by 9
10 if the ones digit is 0
A whole number cannot be divided by 0 .
A variable is a letter or symbol.
It represents a set of numbers in an algebraic expression.
A variable can be used to write an algebraic expression:
" 3 less than a number" can be written as $n-3$.

A variable represents a number in an equation.
A variable can be used to write an equation.
" 4 more than a number is 11 " can be written as $x+4=11$.
An algebraic expression can be evaluated by substituting a number for the variable.
$6 r+3$ when $r=2$ is: $6 \times 2+3=12+3$

$$
=15
$$

A relation describes how the output is related to the input. A relation can be displayed using a table of values or a graph. When points of a relation lie on a straight line, it is a linear relation.

An equation can be solved using tiles.

## What Should I Be Able to Do?

1. Use the divisibility rules to find the factors of 90 .
2. Which of these numbers is 23640 divisible by? How do you know?
a) 2
b) 3
c) 4
d) 5
e) 6
f) 8
g) 9
h) 10
i) 0
3. I am a 3-digit number. I am divisible by 4 and by 9 .
My ones digit is 2 .
I am less than 500.
Which number am I?
Find as many numbers as you can.
4. Draw a Venn diagram with 2 loops. Label the loops "Divisible by 6," and "Divisible by 9."
a) Should the loops overlap? Explain.
b) Write these numbers in the Venn diagram. $\begin{array}{llll}330 & 639 & 5598 & 10217\end{array}$ 229585818712006 How did you know where to put each number?
5. i) Write an algebraic expression for each statement.
ii) Evaluate each expression by replacing the variable with 8 .
a) five less than a number
b) a number increased by ten
c) triple a number
d) six more than three times a number
1.4
6. There are $n$ women on a hockey team.
Write a relation for each statement.
a) the total number of hockey sticks, if each player has 4 sticks
b) the total number of lockers in the dressing room, if there are 3 more lockers than players
c) the total number of water jugs on the bench, if each group of 4 players shares 1 jug
1.5 7. Copy and complete each table. Explain how the Output number is related to the Input number.
a)

| Input | Output |
| :---: | :---: |
| $n$ | $n+13$ |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |

b)

| Input <br> $n$ | Output <br> $5 n+1$ |
| :---: | :---: |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |

c)

| Input | Output |
| :---: | :---: |
| $n$ | $6 n-3$ |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |

8. Use algebra. Write a relation for each Input/Output table.
a)

| Input <br> $n$ | Output |
| :---: | :---: |
| 1 | 12 |
| 2 | 13 |
| 3 | 14 |
| 4 | 15 |

b)

| Input <br> $n$ | Output |
| :---: | :---: |
| 1 | 2 |
| 2 | 7 |
| 3 | 12 |
| 4 | 17 |

9. Match each graph with one of the relations below.
a)

b)

c)

i) $7-3 n$ is related to $n$.
ii) $4 n+3$ is related to $n$.
iii) $n-1$ is related to $n$.
iv) $n+3$ is related to $n$.
v) $3 n-3$ is related to $n$.
vi) $7-n$ is related to $n$.
10. For each relation below:
i) Describe a real-life situation that could be represented by the relation.
ii) Make a table of values.
iii) Graph the relation.
iv) Describe the graph.
v) Write 2 questions you could answer using the graph. Answer the questions.
a) $4+2 m$ is related to $m$.
b) $15-2 d$ is related to $d$.
11. Gerad is paid $\$ 6$ to supervise a group of children at a day camp. He is paid an additional $\$ 2$ per child.
a) Write a relation to show how the total amount Gerad is paid is related to the number of children supervised, $c$.
b) Copy and complete this table of values for the relation.

| $c$ | Amount Paid <br> $(\$)$ |
| :---: | :---: |
| 0 |  |
| 5 |  |
| 10 |  |
| 15 |  |

c) Draw a graph to show the relation. Describe the graph.
d) Use the graph to answer these questions:
i) How much money is Gerad paid when he supervises 25 children?
ii) Gerad was paid $\$ 46$.

How many children did he supervise? Show your work.
12. Suggest a real-life situation that could be represented by this graph.

13. Write an equation for each sentence.
a) A pizza with 15 slices is shared equally among $n$ students. Each student gets 3 slices.
b) Four less than three times the number of red counters is 20.
14. The drum ring of this hand drum is a regular octagon. It has perimeter 48 cm . Write an equation you could use to find the side length of the drum ring.

15. i) Write an equation you can use to solve each problem.
ii) Use tiles to solve each equation.
iii) Draw pictures to represent the steps you took to solve each equation.
iv) Use tiles to verify each solution.
a) Thirty-six people volunteered to canvas door-to-door for the Heart and Stroke Foundation. They were divided into groups of 3 . How many groups were there?
b) A garden has 7 more daffodils than tulips. There are 18 daffodils. How many tulips are there?
c) A sleeve of juice contains 3 juice boxes. Marty buys 24 juice boxes. How many sleeves does he buy?
d) Jan collects foreign stamps. Her friend gives her 8 stamps. Jan then has 21 stamps. How many stamps did Jan have to start with?
16. A number is multiplied by 4 , then 5 is added. The result is 21 . What is the number?
a) Write an equation to represent this situation.
b) Solve the equation to find the number.
c) Verify the solution.

## Practice Test

1. Use the digits 0 to 9 .

Replace the $\square$ in $1621 \square$ so that the number is divisible by:
a) 2
b) 3
c) 4
d) 5
e) 6
f) 8
g) 9
h) 10

Find as many answers as you can.
2. Here are 3 algebraic expressions:
$2+3 n ; \quad 2 n+3 ; \quad 3 n-2$
Are there any values of $n$ that will produce the same number when substituted into two or more of the expressions? Investigate to find out. Show your work.
3. Jamal joined a video club. The annual membership fee is $\$ 25$.

The cost of each video rental is an additional \$2.
a) Write a relation for the total cost when Jamal rents $v$ videos in one year.
b) Graph the relation. How much will Jamal pay when he rents 10 videos? 25 videos?
c) How does the relation change when the cost per video rental increases by \$1?
How much more would Jamal pay to rent 10 videos?
How do you know the answer makes sense?
4. a) Write an equation for each situation.
b) Solve each equation using tiles. Sketch the tiles you used.
c) Verify each solution.
i) There were 22 students in a Grade 7 class. Five students went to a track meet. How many students were left in the class?
ii) Twice the number of dogs in the park is 14. How many dogs are in the park?
iii) Daphne scored the same number of goals in period one, period two, and period three this season. She also scored 4 overtime goals, for a total of 19 goals. How many goals did she score in each period?

## Unit Problem



Two students raised money for charity in a bike-a-thon.
The route was from Lethbridge to Medicine Hat, a distance of 166 km .

## Part 1

Ingrid cycles 15 km each hour.
How far does Ingrid cycle in 1 h ? 2 h ? 3 h ? 4 h ? 5 h ?
Record the results in a table.

| Time (h) |  |  |
| :--- | :--- | :--- |
| Distance (km) | $\square$ |  |

Graph the data.
Graph Time horizontally and Distance vertically.

Write a relation for the distance Ingrid travels in $t$ hours.
How far does Ingrid travel in 7 h ?
How could you check your answer?

Let $t$ hours represent the time Ingrid cycled.
How far does Ingrid cycle in $t$ hours?
Write an equation you can solve to find the time Ingrid took to cycle 120 km .
Solve the equation.

## Part 2

Liam cycles 20 km each hour. Repeat Part 1 for Liam.

## Part 3

How are the graphs for Ingrid and Liam alike? How are they different?

## Check List

## Your work

 should show:$\checkmark$ all tables and graphs, clearly labelled
the equations you wrote and how you solved them
how you know your answers are correct explanations of what you found out

## Part 4

Ingrid's sponsors paid her \$25 per kilometre.
Liam's sponsors paid him \$20 per kilometre.
Make a table to show how much money each student raised for every 10 km cycled.

| Distance <br> $(k m)$ | Money Raised <br> by Ingrid (\$) | Money Raised <br> by Liam (\$) |
| :---: | :---: | :---: |
| 10 |  |  |
| 20 |  |  |
| 30 |  |  |

How much money did Ingrid raise if she cycled $d$ kilometres? How much money did Liam raise if he cycled $d$ kilometres?

Liam and Ingrid raised equal amounts of money.
How far might each person have cycled? Explain.


## Reflect on Your Learning

You have learned different ways to represent a pattern. Which way do you find easiest to use? When might you want to use the other ways?

