- Maggie wrote each number as an equivalent fraction with the same denominator, then placed the fractions on a number line.

Maggie wrote $2\frac{1}{4}$ as an improper fraction: $2\frac{1}{4} = \frac{4}{4} + \frac{4}{4} + \frac{1}{4} = \frac{9}{4}$
Since 12 is a multiple of 3, 4, and 6,
she wrote each fraction with denominator 12.

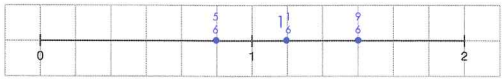
$\frac{9}{4} = \frac{27}{12}$ (×3) $\quad \frac{2}{3} = \frac{8}{12}$ (×4) $\quad \frac{11}{6} = \frac{22}{12}$ (×2)


We can use the placement of the numbers on the line to order the numbers.
The numbers increase from left to right.
So, the order from least to greatest is:
$\frac{8}{12}, \frac{22}{12}, \frac{27}{12}$ or $\frac{2}{3}, \frac{11}{6}, \frac{9}{4}$ or $\frac{2}{3}, \frac{11}{6}, 2\frac{1}{4}$


## Practice

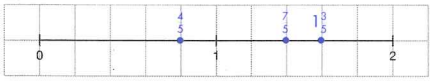
Your teacher will give you copies of number lines for questions 3, 6, and 7.

1. Use 1-cm grid paper.
Draw a 12-cm number line like the one below.




Place these numbers on the line: $\frac{5}{6}, 1\frac{1}{6}, \frac{9}{6}$

2. Use 1-cm grid paper.
Draw a 10-cm number line like the one below.

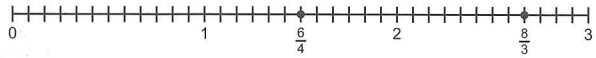


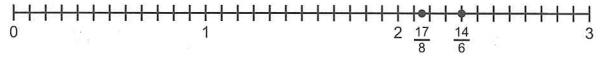

Place these numbers on the line: $1\frac{3}{5}, \frac{7}{5}, \frac{4}{5}$

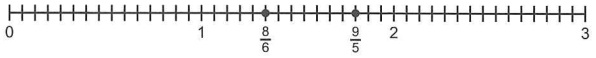


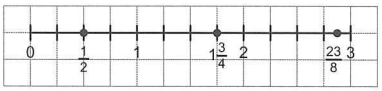
## Sample Solutions

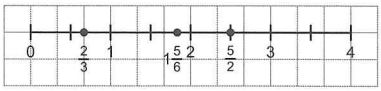
3. Students should write each pair of fractions with the same denominator, then place the fractions on a number line.
For example,

a)



c)



e)



4.



5.


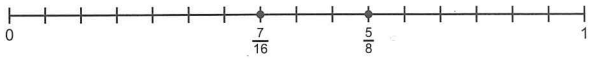

- How can you compare $1\frac{2}{3}$ and $\frac{3}{2}$ without using rods?
*(I can write $1\frac{2}{3}$ as $\frac{5}{3}$. Then I can write equivalent fractions with denominator 6 for $\frac{5}{3}$ and $\frac{3}{2}$.*
*$\frac{5}{3} = \frac{10}{6}$ and $\frac{3}{2} = \frac{9}{6}$*
*Since $\frac{10}{6} > \frac{9}{6}$, then $1\frac{2}{3} > \frac{3}{2}$.)*
- How else could you compare $1\frac{2}{3}$ and $\frac{3}{2}$?
*(I could write $\frac{3}{2}$ as $1\frac{1}{2}$, then compare $1\frac{2}{3}$ and $1\frac{1}{2}$.*
*Since $\frac{2}{3} > \frac{1}{2}$, I know that $1\frac{2}{3} > 1\frac{1}{2}$.)*

Use *Connect* to discuss the three ways of using number lines to compare and order mixed numbers and fractions.

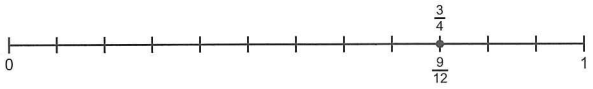
Ask questions, such as:

- What benchmarks did Ella use?
*(0, $\frac{1}{2}$, 1, $1\frac{1}{2}$, 2, $2\frac{1}{2}$, and 3)*
- How can you use Ella's number line to order $2\frac{1}{4}$, $\frac{2}{3}$, and $\frac{11}{6}$ from least to greatest?
*(The number farthest to the left is the least. So, I can read the numbers from left to right.)*
- How can you use Rahim's three number lines to order the numbers?
*(Since the lines are equal in length, the one with the number farthest to the left should be read first.)*
- Why did Maggie rewrite each number so they all had the same denominator?
*(This let her use one number line to find the exact location of each number.)*
- How would you compare $3\frac{4}{7}$ and $2\frac{5}{9}$?
*(First, I look at the whole numbers in each mixed number. Since $3 > 2$, I do not need to compare the fraction parts. Therefore, $3\frac{4}{7} > 2\frac{5}{9}$.)*
- How can you use equivalent fractions to compare $\frac{13}{3}$ and $\frac{17}{6}$?
*(6 is a common multiple of 3 and 6, so I could rewrite $\frac{13}{3}$ with denominator 6; $\frac{26}{6}$.*
*Since $\frac{26}{6} > \frac{17}{6}$, then $\frac{13}{3} > \frac{17}{6}$.)*

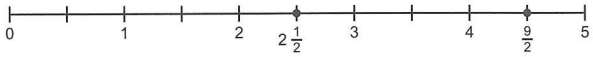
6. a) $\frac{5}{8} > \frac{7}{16}$ because $\frac{5}{6} = \frac{10}{12}$



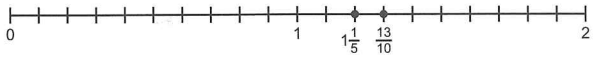

b) $\frac{3}{4} = \frac{9}{12}$ because $\frac{3}{4}$ is equivalent to $\frac{9}{12}$.



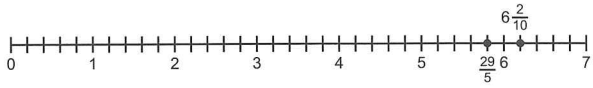

c) $\frac{9}{2} > 2\frac{1}{2}$ because $\frac{9}{2} = 4\frac{1}{2}$



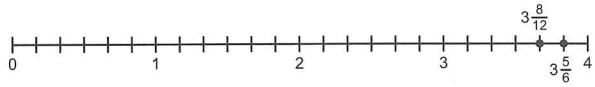

d) $\frac{13}{10} > 1\frac{1}{5}$ because $1\frac{1}{5} = \frac{6}{5} = \frac{12}{10}$



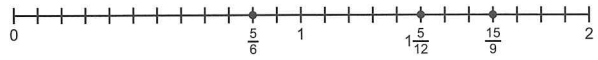

e) $6\frac{2}{10} > \frac{29}{5}$ because $6\frac{2}{10} = \frac{62}{10}$ and $\frac{29}{5} = \frac{58}{10}$




f) $3\frac{5}{6} > 3\frac{8}{12}$ because $3\frac{5}{6} = 3\frac{10}{12}$

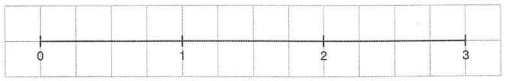



7. a) $\frac{5}{6}$ is the least number because it is the only number less than 1. I wrote $\frac{15}{9}$ as the mixed number $1\frac{6}{9}$, which is equivalent to $1\frac{2}{3}$. Then I wrote $1\frac{2}{3}$ with denominator 12; $1\frac{8}{12}$. So, $1\frac{2}{3} > 1\frac{5}{12}$, which means $\frac{15}{9} > 1\frac{5}{12}$.



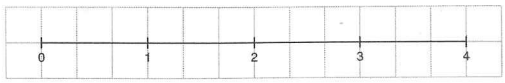

3. Find equivalent fractions so the fractions in each pair have the same denominator.
Place each pair of fractions on a number line.
a) $\frac{8}{3}$ and $\frac{6}{4}$ $\frac{32}{12}$ and $\frac{18}{12}$ b) $\frac{12}{5}$ and $\frac{8}{3}$ $\frac{36}{15}$ and $\frac{40}{15}$
$\frac{56}{24}$ and $\frac{51}{24}$ c) $\frac{14}{6}$ and $\frac{17}{8}$ d) $\frac{11}{10}$ and $\frac{20}{15}$ $\frac{33}{30}$ and $\frac{40}{30}$
e) $\frac{9}{5}$ and $\frac{8}{6}$ $\frac{54}{30}$ and $\frac{40}{30}$ f) $\frac{12}{9}$ and $\frac{11}{5}$ $\frac{60}{45}$ and $\frac{99}{45}$

4. Use 1-cm grid paper.
Draw a number line with the benchmarks 0, 1, 2, and 3 as shown below.




Place these numbers on the number line:
$\frac{1}{2}$, $\frac{23}{8}$, $1\frac{3}{4}$

5. Use 1-cm grid paper.
Draw a number line with the benchmarks 0, 1, 2, 3, and 4 as shown below.




Place these numbers on the number line:
$\frac{5}{2}$, $\frac{2}{3}$, $1\frac{5}{6}$

6. For each pair of numbers below:
- Place the two numbers on a number line.
  Which strategy did you use?
- Which of the two numbers is greater?
  How do you know?

a) $\frac{5}{8}$; $\frac{7}{16}$ $\frac{5}{8} > \frac{7}{16}$ b) $\frac{3}{4}$; $\frac{9}{12}$ $\frac{3}{4} = \frac{9}{12}$ c) $2\frac{1}{2}$; $\frac{9}{2}$ $\frac{9}{2} > 2\frac{1}{2}$
d) $\frac{13}{10}$; $1\frac{1}{5}$ $\frac{13}{10} > 1\frac{1}{5}$ e) $\frac{29}{5}$; $6\frac{2}{10}$ $6\frac{2}{10} > \frac{29}{5}$ f) $3\frac{5}{6}$; $3\frac{8}{12}$ $3\frac{5}{6} > 3\frac{8}{12}$

7. Place the numbers in each set on a number line.
Show how you did it.
List the numbers from least to greatest.

a) $\frac{5}{6}$, $\frac{15}{9}$, $1\frac{5}{12}$ $\frac{5}{6}$, $1\frac{5}{12}$, $\frac{15}{9}$
b) $\frac{9}{4}$, $2\frac{2}{3}$, $\frac{11}{6}$ $\frac{11}{6}$, $\frac{9}{4}$, $2\frac{2}{3}$
c) $\frac{9}{10}$, $\frac{7}{5}$, $\frac{11}{4}$ $\frac{9}{10}$, $\frac{7}{5}$, $\frac{11}{4}$
d) $\frac{10}{3}$, $2\frac{1}{4}$, $\frac{3}{2}$ $\frac{3}{2}$, $2\frac{1}{4}$, $\frac{10}{3}$



- What denominator would you use to write equivalent fractions for $\frac{5}{11}$ and $\frac{18}{12}$? *(132)*
  How did you find your answer?
  *(I multiplied the denominators to find a common multiple of 11 and 12.)*
- Does this way of finding a common multiple always work?
  *(Yes; but it does not always give you the least common multiple of the two denominators.)*

## Practice

Students will need 1-cm grid paper (PM 23) for questions 1, 2, 4, and 5, and Number Lines for Lesson 3 (Master 5.13) for questions 3, 6, and 7.

Have Cuisenaire rods or strips of coloured paper available for students to compare fractions.

### *Assessment Focus: Question 11*

Students realize that they must compare $4\frac{1}{2}$, $\frac{28}{6}$, and $\frac{13}{3}$. Some students will convert $4\frac{1}{2}$ to an improper fraction, find a common denominator for all three improper fractions, then place the three numbers on a number line. They will select the number farthest to the right as representing the most pancakes and the number farthest to the left as representing the least pancakes. Other students may convert $\frac{28}{6}$ and $\frac{13}{3}$ to mixed numbers and place them on a number line along with $4\frac{1}{2}$.

Students can play the Additional Activity *Comparing Fractions* (Master 5.10).

8. Hisa says that $\frac{17}{3}$ is greater than $5\frac{3}{4}$. Is she correct? No
Use pictures, numbers, and words to explain.

Adriel watched a $1\frac{3}{4}$-h movie on TV.
Nadir watched 3 half-hour sitcoms.
Who watched more TV? How do you know? Adriel

10. Justine played a board game for $3\frac{1}{2}$ h.
Marty played the same board game for $\frac{37}{12}$ h.
Who played longer? Justine
Sketch a number line to show how you know you are correct.

11. Ratu, Addie, and Penny cooked pancakes for their school's maple syrup festival in McCreary, Manitoba.
Ratu made $4\frac{1}{2}$ dozen pancakes,
Addie made $\frac{28}{6}$ dozen pancakes,
and Penny made $\frac{13}{3}$ dozen pancakes.
Who made the most pancakes? Addie
Who made the least? Penny
Sketch a number line to show how you know.

McCreary is the maple syrup capital of Manitoba.

12. Florence and her friends Rafael and Bruno race model cars.
Florence's car completed $2\frac{1}{4}$ laps of a track in 1 min.
Rafael's car completed $\frac{8}{3}$ laps of the track in 1 min.
Bruno's car completed $\frac{11}{12}$ laps of the track in 1 min.
Whose car was fastest? How do you know? Rafael's car

13. Use your ruler as a number line.
Visualize placing these fractions on your ruler: $4\frac{3}{5}, \frac{11}{2}, \frac{83}{10}$
Describe where you would place each fraction.
Which fraction is the greatest? The least? $4\frac{3}{5}$, $\frac{83}{10}$

**Reflect**

How do you use a number line to compare fractions and mixed numbers? Include an example.

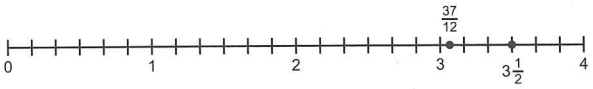


Students' explanations for parts b to d should be similar to that in part a. Number lines should show the numbers in order from least to greatest.

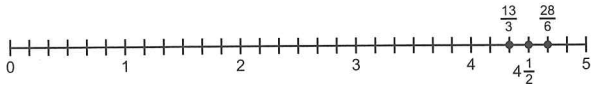
8. $5\frac{3}{4} = \frac{23}{4} = \frac{69}{12}$; $\frac{17}{3} = \frac{68}{12}$
Since $\frac{69}{12} > \frac{68}{12}$, Hisa is not correct.

9. $1\frac{3}{4} = \frac{7}{4}$; $\frac{3}{2} = \frac{6}{4}$; $\frac{7}{4} > \frac{6}{4}$ So, Adriel watched more TV.

10. $3\frac{1}{2} = \frac{7}{2} = \frac{42}{12}$; $\frac{42}{12} > \frac{37}{12}$ So, Justine played longer.




11. $4\frac{1}{2} = \frac{9}{2} = \frac{27}{6}$; $\frac{13}{3} = \frac{26}{6}$ Addie made the most pancakes.
Penny made the least.




12. $2\frac{1}{4} = \frac{9}{4} = \frac{27}{12}$; $\frac{8}{3} = \frac{32}{12}$ Rafael's car was fastest.

13. I would place $4\frac{3}{5}$ at 4.6 cm, $\frac{11}{2}$ at 5.5 cm, and $\frac{83}{10}$ at 8.3 cm. $\frac{83}{10}$ is the greatest and $4\frac{3}{5}$ is the least.

REFLECT: To compare $1\frac{1}{2}$, $\frac{7}{5}$, and $1\frac{3}{5}$, I change each number to an improper fraction with denominator 10: $1\frac{1}{2} = \frac{3}{2} = \frac{15}{10}$; $\frac{7}{5} = \frac{14}{10}$; $1\frac{3}{5} = \frac{8}{5} = \frac{16}{10}$. Then I place each improper fraction on the number line. $\frac{14}{10}$ would come first, $\frac{15}{10}$ second, and $\frac{16}{10}$ third. The number farthest to the left is the least number, and the number farthest to the right is the greatest.

## ASSESSMENT FOR LEARNING

### What to Look For

#### *Conceptual Understanding*

- ✔ Students explain why the same rod must represent one whole when comparing two or more fractions or mixed numbers.
- ✔ Students explain their paper-and-pencil procedures for comparing fractions and mixed numbers.

#### *Procedural Knowledge*

- ✔ Students can convert among proper and improper fractions and mixed numbers.
- ✔ Students can place improper fractions and mixed numbers on a number line.
- ✔ Students can use a variety of methods to compare mixed numbers and improper fractions.

### What to Do If You Don't See It

#### *Check Further*

As students work, ask questions, such as:

- Between which two benchmarks will you place that fraction?
- Is that improper fraction closer to 2 or closer to $2\frac{1}{2}$? How do you know?
- How will writing the numbers as equivalent fractions with the same denominator help you?
- What do you know about numbers to the far left on the number line?

#### *Adjust Instruction*

Write pairs of improper fractions on the board. Have students write equivalent fractions with the same denominator for each pair, then compare the fractions. Use numbers that can easily be written with the same denominator; for example, $\frac{5}{2}$ and $\frac{6}{4}$.