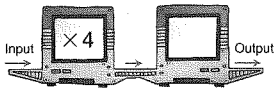
This suggests that the input numbers are multiplied by 4.

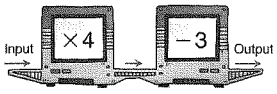


The output increases by 4. Each input must be multiplied by 4.

Look at the input 2.
Multiply by 4.
$2 \times 4 = 8$
But, the output is 5.

Think:

I have 8. To get 5, I subtract 3.
So, −3 goes into the second part of the machine.
$8 - 3 = 5$



I check all the inputs to make sure I have found the correct numbers and the correct operations.

This Input/Output machine multiplies each input by 4, then subtracts 3.
The pattern rule that relates the input to the output is:
Multiply the input by 4.
Then subtract 3.

We can use this rule to predict the output for any input.
For an input of 8, the output should be:
$8 \times 4 - 3 = 29$

We can check this by extending the table.
Add 1 to each input and 4 to each output.

Input	Output	
1	1	
2	5	4
3	9	4
4	13	4
5	17	4
6	21	4
7	25	4
8	29	4

Sample Solutions

1. a) × 7

Input	Output
5	35
6	42
7	49
8	56

Multiply the input by 7.

b) − 11

Input	Output
46	35
45	34
44	33
43	32

Subtract 11 from the input.

c) × 10

Input	Output
10	100
12	120
14	140
16	160

Multiply the input by 10.

d) − 15

Input	Output
300	285
250	235
200	185
150	135

Subtract 15 from the input.

2. a) × 3 − 1

Input	Output
5	14
6	17
7	20
8	23

29

b) × 5 + 4

Input	Output
5	29
6	34
7	39
8	44

54

c) × 2 − 3

Input	Output
7	11
8	13
9	15
10	17

17

d) × 4 + 1

Input	Output
8	33
9	37
10	41
11	45

41

1 each time and the numbers in the output column increase by 4 each time. This suggests that the relationship between columns involves multiplying by 4.

Help students understand that finding a difference of 4 between terms in the output column does not necessarily imply that the pattern rule includes multiplying by 4. For example, if the input numbers increased by 2 each time, the relationship would be different.

Practice

Students may need Colour Tiles or counters and square dot paper (PM 25) for question 5. Two-column charts (PM 19) are required for question 6.

You may wish to point out that the numbers in the input columns in question 1, parts b, c, and d, decrease by 1, increase by 2, and decrease by 50 each time, respectively. For this reason, they cannot use the method in *Connect* to find the pattern rule between the columns. But since these are only single-operation Input/Output machines, students should have no trouble finding the pattern rules.

Some students may have more difficulty with part b of question 4. It might be easier in this case to pretend that the output column is actually the input column, and work backward from there to find the pattern rule.

If students have difficulty with two-operation Input/Output machines, have them add a middle column to their Input/Output tables. This column shows the result of the first step, and then students can see what operation is necessary to get to the second step.

I can extend the tables to check my predictions.

3.

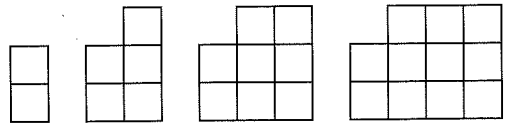


Figure 1 Figure 2 Figure 3 Figure 4

4. a) The pattern rule that relates the input to the output is:
Multiply the input by 3, then add 6.
Output 30: Since I multiply the input by 3, then add 6 to get the output, I subtract 6 from the output, then divide by 3 to get the input.
$30 - 6 = 24$; $24 \div 3 = 8$
The input is 8.
The numbers in the output column increase by 3 each time, so the missing numbers are:
$30 + 3 = 33$ and $33 + 3 = 36$
When the input is 40, I multiply by 3 to get 120, then add 6 to get 126. The output is 126. I could extend the table to check.

(Sample Solutions continue below, right.)

Practice

1. Each table shows the input and output from a machine with one operation. For each table:
- Identify the number and the operation in the machine.
- Continue the patterns.
Write the next 4 input and output numbers.
- Write the pattern rule that relates the input to the output.

a)

Input	Output
1	7
2	14
3	21
4	28

b)

Input	Output
50	39
49	38
48	37
47	36

c)

Input	Output
2	20
4	40
6	60
8	80

d)

Input	Output
500	485
450	435
400	385
350	335

2. Each table shows the input and output from a machine with two operations. For each table:
- Identify the numbers and the operations in the machine.
- Choose 4 different input numbers. Find the output for each input.
- Predict the output when the input is 10. Check your prediction.

a)

Input	Output
1	2
2	5
3	8
4	11

b)

Input	Output
1	9
2	14
3	19
4	24

c)

Input	Output
3	3
4	5
5	7
6	9

d)

Input	Output
4	17
5	21
6	25
7	29

14 Unit 1 Lesson

For example, in question 4a, the numbers in the second column increase by 3 each time. So the middle column could have the title "× 3" as shown:

Input	× 3	Output
5	5 × 3 = 15	21
6	6 × 3 = 18	24
7	7 × 3 = 21	27

It is then easy for students to see that the second step is to add 6.

Assessment Focus: Question 5

Students will find that the numbers in the input column increase by 1 and the numbers in the output column increase by 2, so the method in *Connect* can be used.

In part d, some students will extend the table to find the input that has output 28. Encourage these students to use the pattern rule to check.

(Sample Solutions continued)

b) The pattern rule that relates the input to the output is:
Divide the input by 5, then add 1.
Output 4: Since I divide the input by 5, then add 1 to get the output, I subtract 1 from the output, then multiply by 5 to get the input: $4 - 1 = 3$; $3 \times 5 = 15$.
The input is 15.
The numbers in the output column increase by 1 each time, so the missing numbers are:
$4 + 1 = 5$ and $5 + 1 = 6$
When the input is 40, I divide by 5 to get 8, then add 1 to get 9. The output is 9. I could extend the table to check.

5. a)

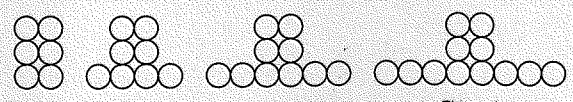


Figure 1 Figure 2 Figure 3 Figure 4

b) Multiply the input by 2, then add 4.
d) I subtracted 4 from 28, then divided by 2.

3. Use the table of values in question 2a.
Draw pictures to show the relationship in the table.

4. Each table shows the input and output from a machine with two operations.
- Find the pattern rule that relates the input to the output.
- Use the pattern rule to find the missing numbers in the table.
- Use the patterns in the columns to check your answers.
- Predict the output when the input is 40. Check your prediction.

a)

Input	Output
5	21
6	24
7	27
✗ 8	30
9	✗ 33
10	✗ 36

b)

Input	Output
0	1
5	2
10	3
15 ✗	4
20	✗ 5
25	✗ 6

2-3

5. You may need Colour Tiles or counters, and dot paper.
a) Use tiles, counters, or pictures to show the relationship in this table. Record your work.
b) Write a pattern rule that relates the input to the output.
c) Predict the output when the input is 9. 22
Extend your pictures to check.
d) Which input has an output of 28? 12
Describe the strategy you used to find out.

Input	Output
1	6
2	8
3	10
4	12

6. a) Draw an Input/Output machine with two operations.
Choose two numbers and two operations for your machine.
b) Choose 5 input numbers. Find the output numbers.
c) Trade tables with a classmate.
Find the pattern rule that relates the input to the output.
Use this pattern to write the next 4 input and output numbers.

Reflect

When you look at an Input/Output table, what strategies do you use to identify the numbers and operations in the machine?

6. a) I drew an Input/Output machine with two operations: $\times 5$ was in the first screen and -2 was in the second screen.

b)

Input	Output
3	13
4	18
5	23
6	28
7	33

c) *Students share their work.*
Next 4 input numbers: 8, 9, 10, 11
Next 4 output numbers: 38, 43, 48, 53

REFLECT: To find the numbers and operations used in an Input/Output table, I check that the input numbers are in order and increasing by 1 each time. I subtract each output number from the number above it to see if the differences are equal. If they are, then this number is what I multiply each input by. I multiply an input number by this constant and decide if any number needs to be added or subtracted to get the output number. I check these numbers and operations to make sure they work for all inputs and outputs.

ASSESSMENT FOR LEARNING

What to Look For

Conceptual Understanding

✔ Students explain that a visual pattern can be shown in a table of values and vice versa.

✔ Students recognize the pattern rule between 2 columns of numbers in a table of values.

Procedural Knowledge

✔ Students can use the pattern rules within columns to help determine the pattern rule between columns.

✔ Students can predict the value of an unknown term using the relationship in a table of values, and verify the prediction.

✔ Students can identify missing terms in a given table of values.

What to Do If You Don't See It

Check Further

As students work on *Practice* question 5, ask:

- Which column of the table of values represents the figure numbers?
- What does the Output column refer to in the visual pattern?
- How does the Input column change? The Output column?
- How can we use the pattern in the columns to help us find the relationship between the two columns?

Adjust Instruction

Students who cannot identify a pattern rule for a table of values should practise with multiplication patterns, then addition or subtraction. Help students calculate the differences of consecutive terms in the output column, which should be the constant that each input is multiplied by. Students can then multiply each input number by this constant and compare the product with the output. This difference is the number that is added or subtracted in the second part of the machine.