

Sample Solutions

1. a)

Input	1	2	3	4	5
Output	9	18	27	36	45

Input to output: Multiply the input by 9.

Input: Start at 1. Add 1 each time.

Output: Start at 9. Add 9 each time.

b)

Input	1	2	3	4	5
Output	13	14	15	16	17

Input to output: Add 12 to the input.

Input: Start at 1. Add 1 each time.

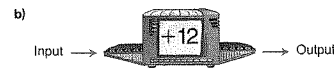
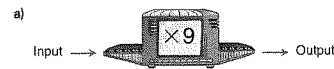
Output: Start at 13. Add 1 each time.

2. a) Input to output: Multiply the input by 6, then add 1.
Input: Start at 2. Add 2 each time.
Output: Start at 13. Add 12 each time.
- b) Input to Output: Add 1 to the input, then multiply by 6.
Input: Start at 2. Add 2 each time.
Output: Start at 18. Add 12 each time.
3. a) The Input/Output machines use the same numbers and operations. They perform the operations in a different order.
- b) For each input, the output in part b is 5 greater than the output in part a.
- c) There is only one output number for each input number. When you multiply a given number by a certain number, there can only be one answer. The same is true for addition. Therefore, you can only get one output number for each input number.

Practice

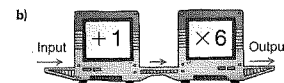
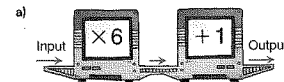
1. For each Input/Output machine:
- Copy and complete the table.
 - Write the pattern rule that relates the input to the output.
 - Write the pattern rule for the input.
 - Write the pattern rule for the output.

Input	Output
1	
2	
3	
4	
5	



2. For each Input/Output machine:
- Copy and complete the table.
 - Write the pattern rule that relates the input to the output.
 - Write the pattern rule for the input.
 - Write the pattern rule for the output.

Input	Output
2	a) 13 b) 18
4	25 30
6	37 42
8	49 54
10	61 66



3. Look at question 2 and your tables.
- How are the Input/Output machines the same? How are they different?
 - How do the output numbers from the two machines compare? Explain.
 - Is it possible to get more than one output number for each input? How do you know?

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Unit 1 Lesson

questions 1 to 5, students will be looking at both "within columns" and "between columns" rules, but in questions 6 and 7, they will be looking at "between columns" rules only.

After you review the second example, ask:

- Could the output be 15? How do you know? (No; the input is multiplied by 2, which results in an even number. Then, we add 6, and the result is still an even number. So, the output is always even.)
- Could the input be 3? How do you know? (Yes; I can use any number as an input number. I just have to multiply it by 2 and add 6 to get the output number.)
- What would the output numbers be for input numbers of 1, 3, and 5? How do you know? (8, 12, and 16; I multiply each input number by 2 and add 6 to get the output number.)

- Which input number has an output of 20? How do you know? (7; I filled in the missing numbers in the input column of the table. Then I filled in their corresponding output numbers in the output column. We calculated the outputs for 1, 3, and 5 as 8, 12, and 16 before, so I continued the output pattern to get an output of 20 for an input of 7.)
- Could you use a different strategy to find the input? Explain. (Yes, I could work backward. The inverse of adding 6 is subtracting 6, and $20 - 6 = 14$. The inverse of multiplying by 2 is dividing by 2, and $14 \div 2 = 7$. So, the input number is 7.)

Ensure students understand that if there is a pattern between the input numbers and the output numbers, then when there is a pattern in the input numbers, there is also a pattern in the output numbers. Each input number and its corresponding output number are related in the same way.

4. Copy and complete this table.
The pattern rule that relates the input to the output is:
Divide the input by 6.
a) Write the pattern rule for the input.
b) Write the pattern rule for the output.

Input	Output
36	6
42	7
48	8
54	9
60	10

5. Copy and complete this table.
The pattern rule that relates the input to the output is:
Divide the input by 3, then subtract 2.
a) Write the pattern rule for the input.
b) Write the pattern rule for the output.

Input	Output
30	8
60	18
90	28
120	38
150	48

6. The pattern rule that relates the input to the output is:
Add 4 to the input. Then divide by 2.
Check the data in the Input/Output table.
Identify any output numbers that are incorrect.
How do you know they are incorrect?
Show your work.

Input	Output
4	4 4
8	4 6
16	10
26	15
30	18 17

7. The pattern rule that relates the input to the output is:
Divide the input by 6, then add 5.
a) Check the data in the Input/Output table.
Identify any output numbers that are incorrect. How do you know they are incorrect?
b) Correct the table.
c) Write 3 more input and output numbers for this pattern rule.
Show your work.

Input	Output
6	6
12	7
30	10
42	12
54	18 14

4. a) Start at 36. Add 6 each time.
b) Start at 6. Add 1 each time.
5. a) Start at 30. Add 30 each time.
b) Start at 8. Add 10 each time.
6. I applied the pattern rule to each input number.
In the output column, 2, 4, and 19 are incorrect.
They should be 4, 6, and 17.
 $4 + 4 = 8$; $8 \div 2 = 4$
 $8 + 4 = 12$; $12 \div 2 = 6$
 $16 + 4 = 20$; $20 \div 2 = 10$
 $26 + 4 = 30$; $30 \div 2 = 15$
 $30 + 4 = 34$; $34 \div 2 = 17$
7. a) I applied the pattern rule to each input number.
 $6 \div 6 = 1$; $1 + 5 = 6$
 $12 \div 6 = 2$; $2 + 5 = 7$
 $30 \div 6 = 5$; $5 + 5 = 10$
 $42 \div 6 = 7$; $7 + 5 = 12$
 $54 \div 6 = 9$; $9 + 5 = 14$
Outputs 2 and 15 are incorrect.

- b) They should be 12 and 14.

c)

Input	Calculation	Output
18	$18 \div 6 = 3$; $3 + 5 = 8$	8
24	$24 \div 6 = 4$; $4 + 5 = 9$	9
36	$36 \div 6 = 6$; $6 + 5 = 11$	11

8. I checked my answers by looking for the pattern in the output column. The pattern rule is: Start at 9. Add 12 each time. All the output numbers fit the pattern.

Practice

Students can use 2-column charts (PM 19) to copy and complete the tables in many of the *Practice* questions.

In many of the *Practice* questions, the numbers in the input column do not follow the pattern 1, 2, 3, In some cases, it is possible to find numbers between the given input numbers. In other cases, such as question 4, it is not. Students at a higher level may notice these differences. Encourage this extra exploration of patterns in tables.

Assessment Focus: Question 7

Students may have difficulty with the fact that the numbers in the input column do not form a pattern. Ensure students understand that in this example, they are looking only at the “between columns” rule, not the “within columns” rule.

Students should recognize that the numbers in the input column must be multiples of 6. Otherwise, students will not get a whole number when they divide the input by 6.

Students who need extra support to complete the Assessment Focus questions may benefit from the Step-by-Step masters (Masters 1.11–1.17).

Students can complete the Additional Activity *Penny Patterns* (Master 1.7).

9. When I was given the input number, I used the pattern rule: Add 5. Then multiply by 3. When I was given the output number, I used the inverse operations and worked backward: I divided by 3, then subtracted 5.
10. Answers will vary. For example:

a) I drew an Input/Output machine with $\times 4$ in the first screen and $+3$ in the second screen.

b)

Input	Output
5	23
10	43
15	63
20	83
25	103

c) Students share their work.

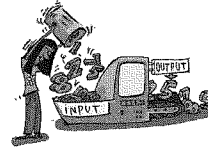
REFLECT: There are 1000 mm in 1 m. So, to convert millimetres to metres, I divide by 1000. The machine would take the input number (in millimetres) and divide it by 1000 to get the output number (in metres). The machine would contain one operation: $\div 1000$

8. The pattern rule that relates the input to the output is: Multiply the input by 4. Then subtract 3. Find the missing numbers in the table. How can you check your answers?

Input	Output
3	9
6	\swarrow 21
9	\swarrow 33
12	45
15	\swarrow 57

9. The pattern rule that relates the input to the output is: Add 5 to the input. Then multiply by 3. Find the missing numbers in the table. What strategies did you use?

Input	Output
2	21
5	\swarrow 30
\swarrow 8	39
11	\swarrow 48
\swarrow 14	57
\swarrow 17	66



10. a) Draw an Input/Output machine with two operations. Choose two numbers and two operations for your machine.
 b) Choose 5 input numbers. Find the output numbers.
 c) Erase 2 input numbers and 2 output numbers. Each row must have at least one number. Trade tables with a classmate. Trade pattern rules that relate the input to the output. Find your classmate's missing numbers.

Reflect

Suppose you want to make an Input/Output machine to convert millimetres to metres. Describe what your machine would look like.

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Unit 1 Lesson 1

ASSESSMENT FOR LEARNING

What to Look For

Conceptual Understanding

- ✓ Students explain that an Input/Output machine is used to create a pattern.

Procedural Knowledge

- ✓ Students can identify, extend, and create patterns with Input/Output machines.
- ✓ Students can identify erroneous data in a table of input and output numbers.
- ✓ Students can describe the pattern within each column of a table of values.
- ✓ Students can generate values in one column of a table of values given values in the other column and a pattern rule.

What to Do If You Don't See It

Check Further

As students work, ask:

- What are the differences in consecutive input numbers?
- What are the differences in consecutive output numbers?
- How can you use these differences to write pattern rules for the input and output numbers?
- How can you use the pattern rules to extend the table?

Adjust Instruction

Students who have difficulty identifying number patterns may benefit from creating their own patterns. Have students work independently to create and extend different types of number patterns. They can trade patterns with a classmate, and identify each other's patterns.